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A PARTITIONED APPROACH TO MODEL TSUNAMI IMPACT ON COASTAL PROTECTIONS

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Abstract. *In this paper we consider the modelling of a tsunami impacting on coastal protections. The phases of generation and propagation across the ocean are not considered here (see for example [2, 6]), only the near-shore phase, which is modified through the influence of structural effects. Viscous effects are considered near the structures.*

This problem is intrinsically a multi-physics one, where different spatial discretization methods, time integration schemes, time steps and even softwares are used for the sub-problems. In this context, partitioned approaches which try to preserve the independence between each component make perfect sense.

1 INTRODUCTION

The numerical computation of coupled problems is characterized nowadays by more and more complex simulations, with respect to the presence of different time scales characterizing each coupled sub-problem. Despite this complexity, one is required by the practical applications to master the interaction between different fields of physics (e.g. thermomechanics [5], fluid-structure interactions or coupling of mechanics with physical-chemistry…), thus pushing the coupled problems into the mainstream of current research.

In this work, we focus on the modelling of the impact of tsunami waves on coastal protections (see Fig. 1); such a problem is intrinsically a multi-physics one, and its solution requires the use of complex numerical tools:

- The propagation of tsunami waves in the near-shore zone, which is a fully non-linear problem in the domain under consideration (Section 2.2);
- The effect of viscous damping which cannot be neglected near the beach or near the coastal engineering protections (Section 2.1);
- The mechanical behavior of the structures: very complex non-linear laws that represent at the macro-scale the behavior of engineering materials are presently being implemented on Finite Element Method (FEM) based codes (Section 2.3).

The coupling between the sub-problems is described in Section 3.

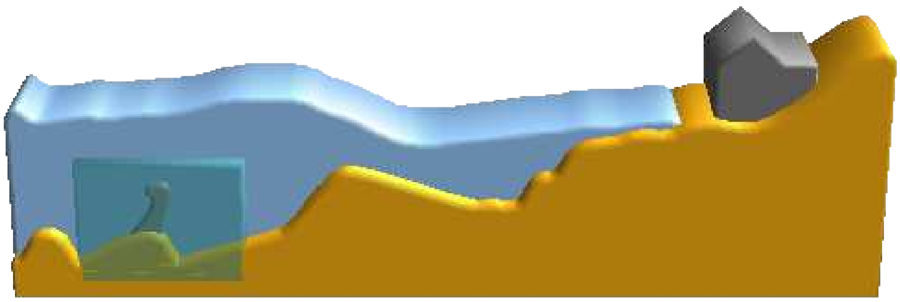

Figure 1: Coupled problem under consideration : only the left part (Wave propagation, viscous fluid and structure)

2 THE DIFFERENT SUBPROBLEMS

2.1 INCOMPRESSIBLE NEWTONIAN FLUID

Let Ω be an open of $\mathbb{R}^3$ filled with a fluid. A point in $\mathbb{R}^3$ is denoted by $\mathbf{x} = (x_1, x_2, x_3)$. The mass balance equation leads to the following classical so-called continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \tag{1}$$

where ρ denotes the density of the fluid, $\mathbf{v}$ the velocity field in Ω and $\nabla\cdot$ the divergence operator. Adding the hypothesis of incompressibility, the above equation becomes

$$\nabla \cdot \mathbf{v} = 0 \tag{2}$$

Applying Newton's second law and taking into account the continuity equation gives the following momentum equation:

$$\rho\frac{\mathrm{D}\mathbf{v}}{\mathrm{D}t} - \nabla\cdot\boldsymbol{\sigma} = \rho\mathbf{f} \tag{3}$$

where $\boldsymbol{\sigma}$ denotes a 3×3 symmetric tensor known as the stress tensor and $\mathbf{f}$ the volume forces applied to the fluid in Ω. The derivative $\frac{\mathrm{D}}{\mathrm{D}t}$ is sometimes called the total (or material) time derivative operator.

To complete this description one needs a *law of behavior* to link the velocity $\mathbf{v}$ and the stress tensor $\boldsymbol{\sigma}$. In the case of Euler flows, the stress tensor reduces to $\boldsymbol{\sigma} = -p\mathbf{I}$, but for viscous flows, the stress tensor can be written as:

$$\boldsymbol{\sigma} = \tilde{\boldsymbol{\sigma}}(\mathbf{D}) - p\mathbf{I}, \tag{4}$$

where the viscous stress tensor $\tilde{\boldsymbol{\sigma}}$ is a function of the symmetric deformation rate tensor $\mathbf{D}$:

$$\mathbf{D} = \frac{1}{2}\left(\nabla\mathbf{v} + (\nabla\mathbf{v})^T\right) \tag{5}$$

One can make an analogy with Hooke's law for the linear case leading to the Newtonian fluid approximation:

$$\tilde{\boldsymbol{\sigma}} = 2\mu\left(\mathbf{D} - \frac{1}{3}(\nabla\cdot\mathbf{v})\mathbf{I}\right) + \kappa(\nabla\cdot\mathbf{v})\mathbf{I} \tag{6}$$

The first term, proportional to the dynamic viscosity μ, models the viscous effects associated to the volume invariant deformations, while the second term, proportional to the volume viscosity κ, models the viscous effects associated to volume dilatation. In general, κ is small compared to μ.

The problem we just described is solved by the *IcoFoam* solver of OpenFoam: a transient solver for the laminar flow of incompressible Newtonian fluids. This solver is based on an Eulerian description and an explicit time integration. Like other Volume Of Fluid (VOF) solvers, it is very expensive and must be limited to a small area. In our case, the area considered is near the obstacle, where viscosity effects can be important. In the most parts of the fluid domain, viscosity effects can be neglected and we use the method described in the next subsection.

2.2 NON-LINEAR WAVE PROPAGATION

To simulate and analyze this phenomenon, we generate solitary waves in a 3D numerical wave tank (NWT). The NWT solves fully non-linear potential flow equations with a free surface, using a high-order boundary element method and a mixed Eulerian-Lagrangian time updating. Some numerical aspects of the NWT have been improved recently, such as the accurate computation of higher-order derivatives on the free surface and the implementation of a fast multipole algorithm in the spatial solver. The former has allowed the accurate simulation of 3D overturning waves and the latter has led to at least a one-order of magnitude increase in the NWT computational efficiency. This made it possible to generate finely resolved 3D overturning waves and analyze their geometry and kinematics (for more details see [3]).

To be more precise, the surface wave problem consists in solving Laplace's equation for the velocity potential $\mathbf{v} = \nabla\phi$ in the whole fluid domain $\Omega(t)$:

$$\Delta\phi = 0, \qquad \forall\mathbf{x} = (x_1, x_2, x_3) \in \Omega(t) \tag{7}$$

Green's second identity transforms this equation into a boundary integral equation:

$$\alpha(\mathbf{x}_l)\phi(\mathbf{x}_l) = \int_{\partial\Omega(t)} \left[\frac{\partial\phi}{\partial\mathbf{n}}(\mathbf{x})G(\mathbf{x},\mathbf{x}_l) - \phi(\mathbf{x})\frac{\partial G}{\partial\mathbf{n}}(\mathbf{x},\mathbf{x}_l)\right] \mathrm{d}\partial\Omega(t) \tag{8}$$

The domain $\Omega(t)$ is bounded above by a moving free surface (interface between air and water) and below by a fixed solid boundary. The free surface is represented by $F : (\mathbf{x}, t) \rightarrow \eta(x_1, x_2, t) - x_3 = 0$ and the bottom is given by $h(x_1, x_2) + x_3 = 0$. G denotes the 3D free-space Green's function. The free surface η is the new fundamental unknown of the problem. Two boundary conditions are required along the free surface:

- The kinematic condition

$$\frac{\mathrm{D}F}{\mathrm{D}t} = \frac{\mathrm{D}\eta}{\mathrm{D}t} - \frac{\mathrm{D}x_3}{\mathrm{D}t} = \frac{\partial\eta}{\partial t} + \nabla\phi\cdot\nabla\eta - \frac{\partial\phi}{\partial x_3} = 0 \tag{9}$$

- The dynamic condition stating that the normal stress at the free surface is given by the difference in pressure. Bernoulli's equation evaluated on the free surface yields

$$\frac{\partial\phi}{\partial t} + \frac{1}{2}\left\|\nabla\phi\right\|^2 + g\eta = 0 \tag{10}$$

The only condition to enforce at the bottom is the kinematic condition:

$$\nabla\phi\cdot\nabla h + \frac{\partial\phi}{\partial x_3} = 0 \tag{11}$$

As seen by the equations above, the problem is defined only on the boundary. The spatial discretization is performed through a Boundary Element Method (BEM) problem. The time integration is explicit [3].

Notice that wave breaking cannot be considered, thus showing the limits of BEM. As our goal is to measure the influence of structural effects on the damping of ocean waves, it is not necessary to model the breaking waves since the wave energy can be computed before this event.

2.3 MECHANICAL PART

The mechanical part of the problem cannot be considered as an open problem. We consider a very classical FEM formulation. Such a classical approach is very reliable, relatively inexpensive, and allows the use of very complex behavior laws with internal variables. For more details, we refer to [4, 9].

Notice that the FEM formulation is a Lagrangian one. Moreover, as the internal properties in such formulation are of great importance, the time integration relies on implicit schemes.

3 COUPLING THE SUBPROBLEMS

In this Section, we describe shortly the coupling of formulations described in Section 2.1, 2.2 and 2.3.

3.1 SOFTWARE IMPLEMENTATION

The different parts of our problem have been solved by different research teams, and one of the goals of our work is to show the possibility of re-using existing codes in a multi-physics context.

- The NWT problem is solved by a Fortran code using a C program to implement the fast multipole algorithm [3].
- The VOF code is OpenFoam, a general C++ library to solve fluid problems.
- The mechanical part is solved by FEAP, a FEM code written in Fortran [9].

Each program is considered as a component. The communication between the above components is performed by the Component Template Library (CTL) [7, 8], and each component can be launched on the same personal computer or processor, on different processors of the same cluster, or also on different machines through network communication.

3.2 COUPLING ALGORITHM

The CTL allows communication between the components. As said in Section 2.2, weakly coupled strategies are proposed when BEM codes become unavailable. Such strategies, where the results from BEM codes are given to other software without any "come-back", present the advantages of simplicity and small cost, but even if the error propagation at each data exchange can be estimated, it can lead to the phenomenon of over- or under-estimated physical instabilities [1, 7].

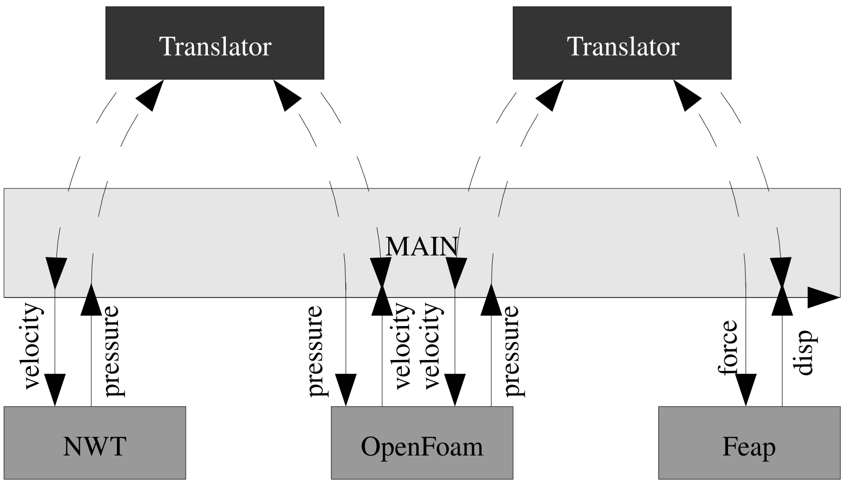

Figure 2: Software implementation and data exchange between components

Once these tools are given, it remains the question of the coupling algorithm: which data (physical quantities) are to be exchanged, and in which order ? We consider independent time integration solvers in a window (i.e. $t \in [T_n, T_{n+1}]$). For this reason not only the value at the synchronization points T_n or T_{n+1}, but also the interpolated evolution of the considered variables on the whole window must be exchanged. Furthermore the strongly coupled problem under consideration requires an error evaluation in order to know if there is convergence. For us this error is based on the coupling data.

4 CONCLUSION

In this paper we have explored a partitioned strategy to solve strongly coupled fluid-structure interaction problems under a moving free surface. Such an approach arises naturally when one wants to use methods and softwares developed independently to solve each sub-problem ; in this strategy new computing tools, like CTL, and coupling algorithms that preserve the independence of sub-problems and numerical stability for the whole problem become the key point.

A useful direction for future research is a multi-scale modeling of tiny obstacles that can represent, for instance, the sea vegetation that recent events in South-Asia have shown to be of great importance.

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