Numerical simulation of powder snow avalanches

DENYS DUTYKH[1]
CR CNRS

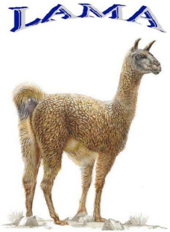
[1]CNRS-LAMA, Université de Savoie
Campus Scientifique
73376 Le Bourget-du-Lac, France

« Variabilité des phénomènes et des milieux naturels »

Acknowledgements

Collaborators :

- Didier Bresch : DR CNRS, LAMA
- Céline Acary-Robert : IR UDS, LAMA



Special thanks to :

- Frédéric Dias : Professor, CMLA (ENS de Cachan)
- Jean-Michel Ghidaglia : Professor, CMLA (ENS de Cachan)

Outline

1. Physical context
2. Mathematical modelling
3. Numerical results
4. Perspectives and conclusions

Outline

1 Physical context

2 Mathematical modelling

3 Numerical results

4 Perspectives and conclusions

Powder-snow avalanches

Large scale turbidity currents descending slopes at high velocities

Characteristic values :

Height : $H \sim 100$ m

Front velocity : $U_f \sim 100$ m/s

Density : 4 kg/m$^3 \leq \rho \leq 25$ kg/m^3

Powder-snow avalanches

Large scale turbidity currents descending slopes at high velocities

Characteristic values :

Height : $H \sim 100$ m

Front velocity : $U_f \sim 100$ m/s

Density : 4 kg/m$^3 \leq \rho \leq 25$ kg/m^3

FIG.: Protecting wall in armed concrete at Taconnaz

Outline

1. Physical context
2. Mathematical modelling
3. Numerical results
4. Perspectives and conclusions

Mathematical models

Classification of various existing approaches

- Probabilistic / Deterministic models
- Discrete
  - granular material
  - cellular automata
- Continuous
  - 0D : conservation laws expressed as ODEs (Kulikovskiy & Sveshnikova (1970), Beghin (1979))
  - 2DH : depth integrated equations (Savage-Hutter)
  - 3D : Free-surface flow
  - 3D : Two-fluid simulations (DNS)

《 + 》: Both fluids are resolved

《 + 》: Flow structure is completely determined

《 - 》: Simulations are expensive

- Limited to relatively simple situations

Mathematical model

J. Etienne, P. Saramito, E.J. Hopfinger. Numerical simulations of dense clouds on steep slopes : Application to powder-snow avalanches. Annals of Glaciology, 38, (2004)

- Flow of two miscible fluids : ϕ – the volume fraction

$$\rho = \phi\rho^+ + (1-\phi)\rho^-, \quad \mu = \phi\rho^+\nu^+ + (1-\phi)\rho^-\nu^-$$

- Flow is incompressible

$$\nabla \cdot \vec{u} = 0$$

- Two fluids share the same velocity $\vec{u}$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u} + \frac{\nabla p}{\rho} = \vec{g} + \frac{1}{\rho}\nabla\cdot\big(2\mu\mathbb{D}(\vec{u})\big)$$

- Mixing is modeled by Fick's law

$$\frac{\partial \phi}{\partial t} + \vec{u}\cdot\nabla\phi = \nabla\cdot(\nu_f\nabla\phi)$$

Kinetic energy balance equation

$$\frac{d}{dt}\frac{1}{2}\int_\Omega \rho|\vec{u}|^2\,d\Omega = \int_\Omega \frac{|\vec{u}|^2}{2}\nabla\cdot(\nu_f\nabla\rho)\,d\Omega + \int_\Omega \rho\vec{g}\cdot\vec{u}\,d\Omega - \int_\Omega 2\mu|\mathbb{D}(\vec{u})|^2\,d\Omega$$

Important remark :

- When we add diffusion in mass conservation, momentum equation has to be corrected

Energetically consistent model :

$$\frac{\partial\vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u} - \nu_f\nabla\log\rho\cdot\nabla\vec{u} + \frac{\nabla p}{\rho} = \vec{g} + \frac{1}{\rho}\nabla\cdot\left(2\mu\mathbb{D}(\vec{u})\right)$$

$$\frac{d}{dt}\frac{1}{2}\int_\Omega \rho|\vec{u}|^2\,d\Omega = \int_\Omega \rho\vec{g}\cdot\vec{u}\,d\Omega - \int_\Omega 2\mu|\mathbb{D}(\vec{u})|^2\,d\Omega$$

Governing equations

Classical gravity current scaling

- Dimensionless form :

$$\begin{aligned} \nabla\cdot\vec{u} &= 0, \\ \frac{\partial\phi}{\partial t}+\vec{u}\cdot\nabla\phi &= \nabla\cdot\left(\frac{1}{\mathrm{ReSc}}\nabla\phi\right), \\ \frac{\partial\vec{u}}{\partial t}+(\vec{u}\cdot\nabla)\vec{u}-\frac{1}{\mathrm{ReSc}}\nabla\log\rho\cdot\nabla\vec{u}+\frac{\nabla p}{\rho} &= \vec{g}+\frac{1}{\rho}\nabla\cdot\left(\frac{1}{\mathrm{Re}}2\mu\mathbb{D}(\vec{u})\right) \end{aligned}$$

$$\rho := \phi+(1-\phi)\frac{\rho^-}{\rho^+}, \quad \mu := \phi+(1-\phi)\frac{\rho^-}{\rho^+}\frac{\nu^-}{\nu^+}$$

Four scaling parameters to be respected :

Reynolds Re ; Schmidt Sc ; Atwood At ; viscosity ratio $\frac{\nu^-}{\nu^+}$

Outline

1. Physical context
2. Mathematical modelling
3. Numerical results
4. Perspectives and conclusions

Numerical methods

Finite volumes method

OpenFOAM — *The Open Source CFD Toolbox*

Our solver is based on :

- `twoLiquidMixingFoam`
- This standard solver was **modified** to include the extra term

- Structured meshes
- Second-order upwind finite volumes scheme
- Implicit time discretization

Remark :

Excellent local conservative properties

Conservation issues with FEM

J. Etienne. PhD thesis, INPG, Grenoble (2004)

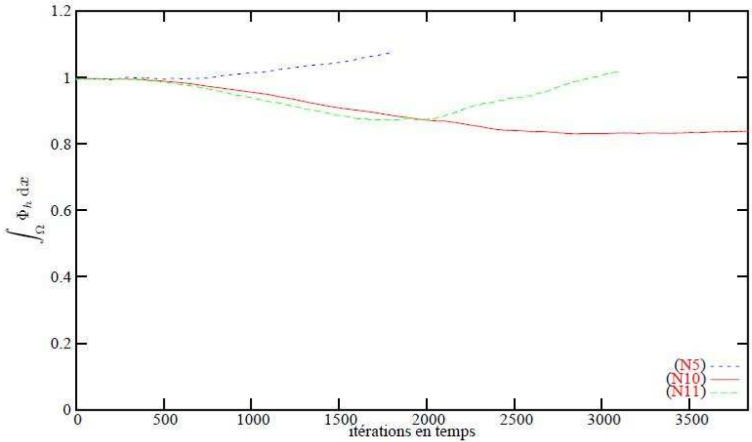


FIG. 31 – *Conservation approchée de la masse.*

Conservation issues with FEM

J. Etienne. PhD thesis, INPG, Grenoble (2004)

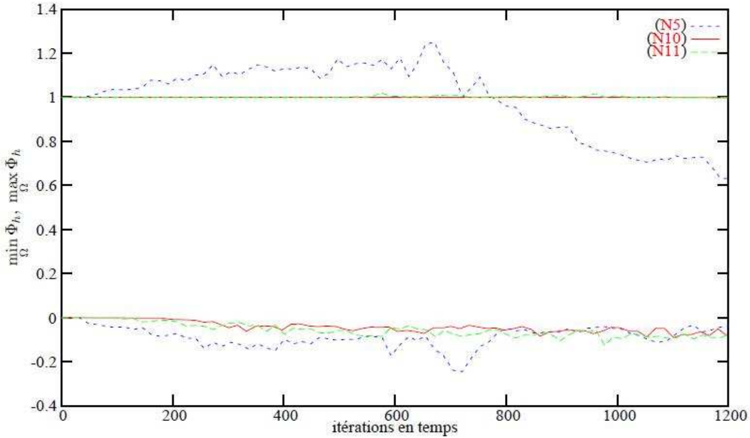
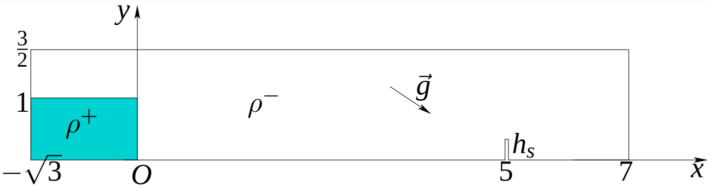


FIG. 32 – *Principe du maximum approché : maximum de l'écart de Φ_h avec ses bornes supérieures et inférieures théoriques.*

Sliding mass test-case

Classical lock-exchange type flow

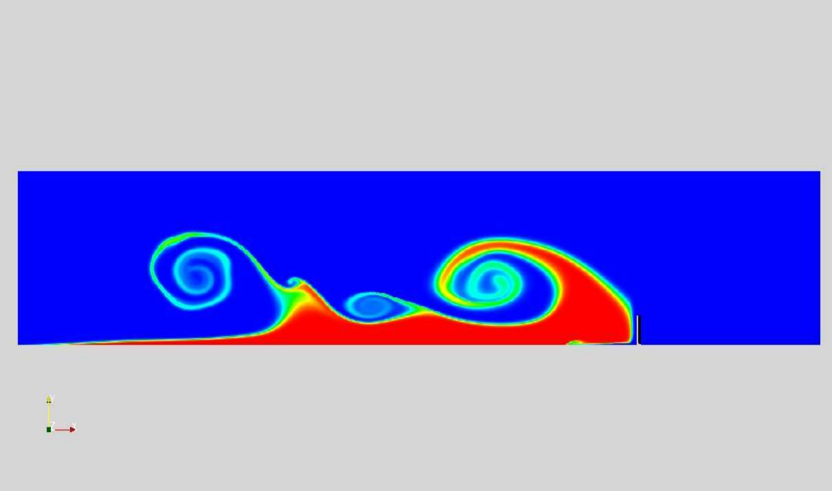


parameter	*value*
gravity acceleration g, m/s^2	9.8
slope, θ	$32°$
Reynolds number, Re	10^5
Schmidt number, Sc	0.5
friction parameter, α	0.3
initial mass height, h_0	1
obstacle height, h_s	$0.25h_0$
obstacle thickness	$0.05h_0$

parameter	*value*
ρ^+, kg/m^3	4
ρ^-, kg/m^3	1
ν^+, m^2/s	10^{-5}
ν^-, m^2/s	10^{-5}

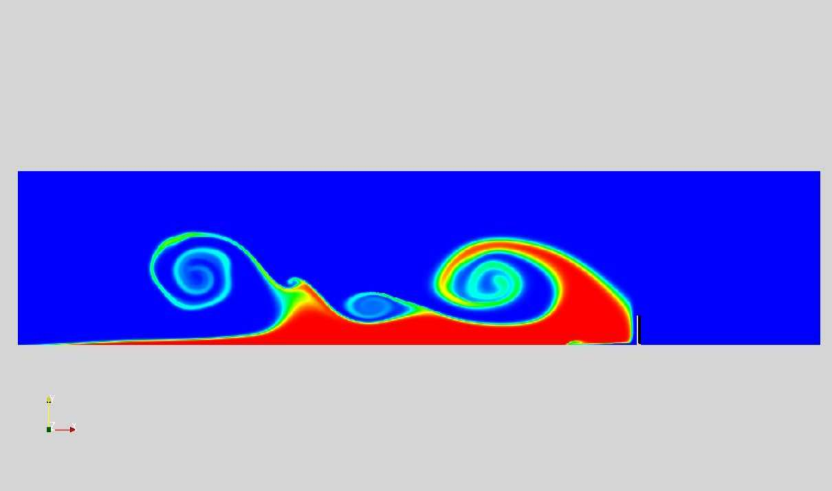
Sliding mass test case

Interaction with a « small » obstacle : $h = 0.25h_0$



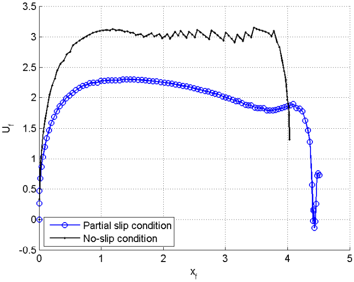
Sliding mass test case

Interaction with a « big » obstacle : $h = 0.6h_0$

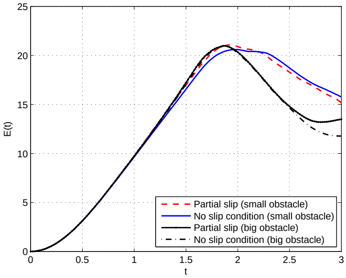


Computational results

Front velocity and kinetic energy



(a) Front velocity



(b) Kinetic energy evolution with time

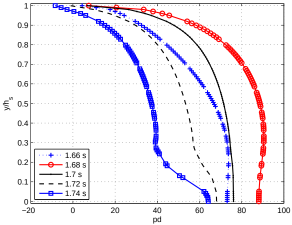
Reference :

D. Dutykh, C. Acary-Robert, D. Bresch. *Numerical simulation of powder-snow avalanche interaction with an obstacle*. Submitted to Applied Mathematical Modelling (2009)

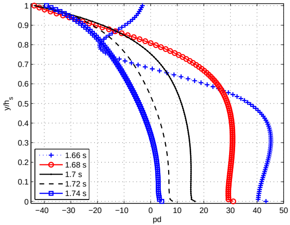
Empiric law (Beghin & Closet, 1990) :

$$p_d = \frac{K}{2} K_a(z) \bar{\rho} U_f^2, \quad K_a(z) = \begin{cases} 10, & z < 0.1h, \\ 19 - 90z, & 0.1h \le z \le 0.2h, \\ 1, & z > 0.2h, \end{cases}$$

Computation results :



(c) $h_s = 0.25h_0$



(d) $h_s = 0.6h_0$

Outline

1. Physical context
2. Mathematical modelling
3. Numerical results
4. Perspectives and conclusions

Conclusions and perspectives

Conclusions :

- We presented a two-fluid energetically *consistent* model
- Efficient and locally *conservative* scheme
- Allows for accurate computation of...
  - impact pressures
  - front position and velocity
  - kinetic energy loss
  - etc.

Perspectives :

- Comparison with experimental data
- Compressible effects
  - $\mathrm{Ma} \approx 0.3$
- Realistic boundary conditions at the soil

Thank you for your attention !

http://www.lama.univ-savoie.fr/~dutykh