Tsunami wave energy

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Nonlinear Coherent Wave Structures in Fluids

Outline

Outline

Energy of tsunamis

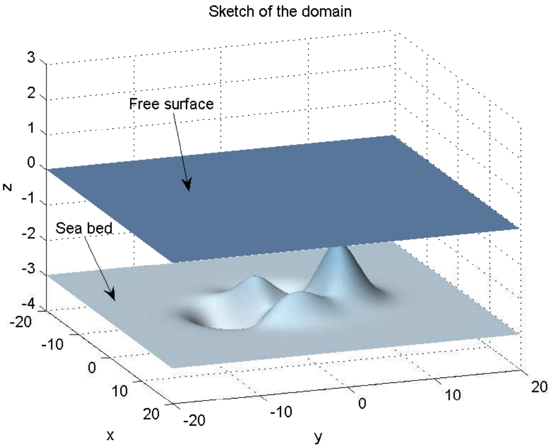
Existing literature (this list is not exhaustive)

1. S. Tinti & E. Bortolucci. *Energy of Water Waves Induced by Submarine Landslides*. Pure appl. geophys. 157 (2000)
2. Z. Kowalik et al. *The Tsunami of 26 December 2004 : Numerical Modeling and Energy Considerations*. 22nd IUGG International Tsunami Symposium (2005)
3. T. Murty et al. *Leakage of the Indian Ocean Tsunami Energy into the Atlantic and Pacific Ocean*. J. Canadian Association of Exploration Geophysicists (2005)
4. A. Velichko et al. *Amplitude-energy characteristics of tsunami waves for various types of seismic sources generating them*. Physical Oceanography (2002)

Traditional approach

Approaches to generation

Put coseismic displacements directly on the free surface and let it propagate :



Traditional approach

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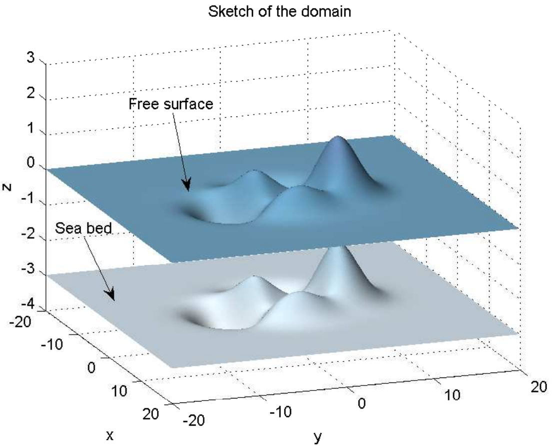


Tsunami generation process

Different scenarios of bottom motion

We consider the following situations :

1. Passive generation
   - $\Rightarrow$ Initial Value Problem (IVP)
2. Instantaneous seabed deformation

$$h(\vec{x}, t) = h_0(\vec{x}) + \mathcal{H}(t)\zeta(\vec{x})$$

3. Exponential scenario

$$h(\vec{x}, t) = h_0(\vec{x}) + (1 - e^{-\alpha t})\zeta(\vec{x})$$

Remark

Passive generation is not equivalent to instantaneous seabed motion

Analytical solutions

Linearized water waves : Cauchy-Poisson problem

- Free surface elevation

$$\eta(\vec{x}, t) = \frac{1}{(2\pi)^2} \iint\limits_{\mathbb{R}^2} \frac{\widehat{\zeta}(\vec{k}) e^{i\vec{k}\cdot\vec{x}}}{\cosh(|\vec{k}|h)} \frac{1}{2\pi i} \int\limits_{\mu - i\infty}^{\mu + i\infty} \frac{s^2 \mathrm{T}(s)}{s^2 + \omega^2} e^{st} ds \, d\vec{k}$$

- Velocity potential

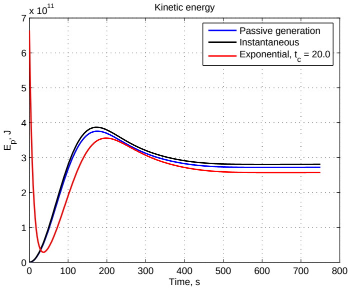
$$\hat{\phi}(\vec{x}, z, t) = \frac{g s \widehat{\zeta}(\vec{k}) \mathrm{T}(s)}{\cosh(|\vec{k}|h)(s^2 + \omega^2)} \left(\frac{s^2}{g|\vec{k}|} \sinh(|\vec{k}|z) - \cosh(|\vec{k}|z) \right)$$

- Compute the energies

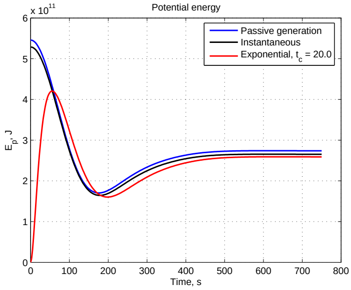
$$K = \frac{1}{2} \int\limits_{-h}^{\eta} \iint\limits_{\mathbb{R}^2} |\nabla \phi|^2 \, d\vec{x} dz \quad \Pi = \frac{g}{2} \iint\limits_{\mathbb{R}^2} \eta^2 \, d\vec{x}$$

Energy analysis of generation process - I

Kinetic and potential energies



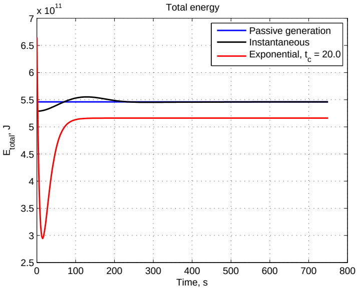
(a) Kinetic energy



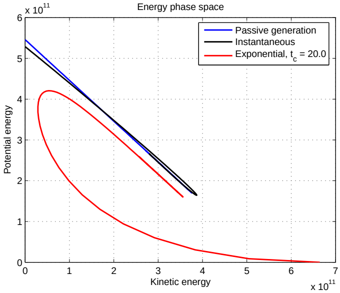
(b) Potential energy

Energy analysis of generation process - II

Total energy and phase space



(c) Total energy



(d) Energy phase space

Outline

Energy equation derivation for long waves

Starting point : Euler equations with free surface

After integrating energy equation over the depth :

$$E_t + \nabla \cdot \Phi + P = 0$$

- Total energy

$$E = \int_{-h}^{\eta} \Big(\frac{1}{2}\rho(|\vec{u}|^2 + w^2) + \rho g z \Big) dz$$

- Energy flux

$$\Phi = \int_{-h}^{\eta} \vec{u} \Big(\frac{1}{2}\rho(|\vec{u}|^2 + w^2) + p + \rho g z \Big) dz$$

- Energy input term

$$P = p_s \eta_t + p_b h_t$$

Energy equation derivation for long waves

Switching to dimensionless quantities

After integrating energy equation over the depth :

$$E_t + \varepsilon \nabla \cdot \Phi + \varepsilon P = 0$$

- Total energy

$$E = \int\limits_{-h}^{\varepsilon\eta} \Big(\frac{\varepsilon}{2}(\varepsilon |\vec{u}|^2 + \frac{\varepsilon}{\mu^2} w^2) + z \Big) dz$$

- Energy flux

$$\Phi = \int\limits_{-h}^{\varepsilon\eta} \vec{u} \Big(\frac{\varepsilon}{2}(\varepsilon |\vec{u}|^2 + \frac{\varepsilon}{\mu^2} w^2) + \varepsilon p + z \Big) dz$$

- Energy input term

$$P = \varepsilon p_b \zeta_t$$

Governing equations - I

NSW2E : Nonlinear Shallow Water Equations with Energy

- Mass conservation

$$H_t + \nabla \cdot (H\vec{u}) = 0$$

- Momentum conservation

$$(H\vec{u})_t + \nabla \cdot \left(H\vec{u} \otimes \vec{u} + \frac{g}{2}H^2\right) = gH\nabla h$$

- Energy equation

$$(HE)_t + \nabla \cdot \left((HE + \rho\frac{g}{2}H^2)\vec{u}\right) + \rho g H \zeta_t = 0$$

Governing equations - II

Boussinesq equations with energy

- Mass conservation

$$H_t + \nabla \cdot (H\vec{u}) = 0$$

- Momentum conservation

$$(H\vec{u})_t + \nabla \cdot \big(H\vec{u} \otimes \vec{u} + \frac{g}{2}H^2\big) + \Big(\frac{h^3}{6}\nabla\big(\nabla \cdot (\frac{H\vec{u}}{d})\big) - \frac{h^2}{2}\nabla\big(\nabla \cdot (H\vec{u})\big)\Big)_t = gH\nabla h$$

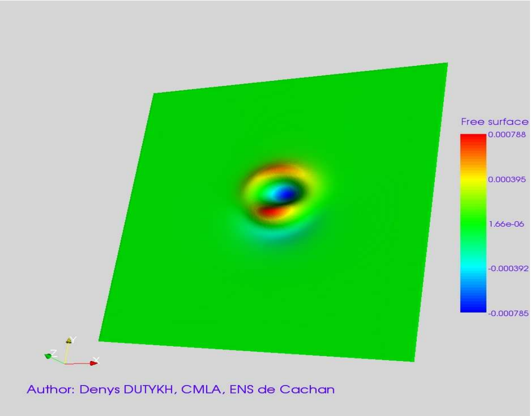
- Energy equation

$$(HE)_t + \nabla \cdot \big((HE + \rho\frac{g}{2}H^2)\vec{u}\big) + \mathcal{O}(\varepsilon\mu^2) + \rho g H \zeta_t = 0$$

Outline

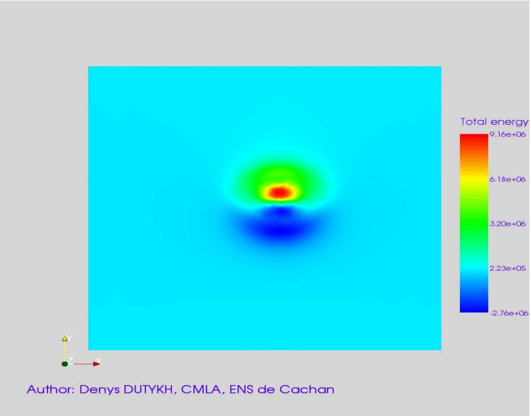
Active generation of a tsunami wave - I

Free surface elevation



Active generation of a tsunami wave - II

Total energy density plot



Energy reconstruction

Computation of the total energy from NSWE solutions

1. We solve NSWE $\Rightarrow (\eta, h, \vec{u})$
2. Using this information, we approximate the energy density :

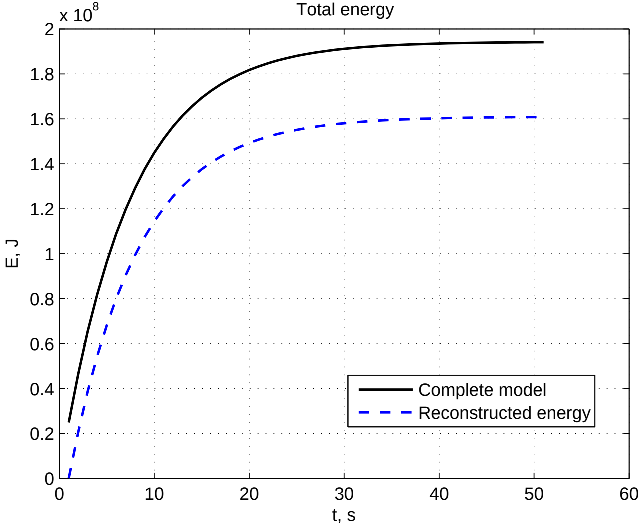
$$E \approx (h+\eta)\Big(\frac{1}{2}\rho|\vec{u}|^2 + \frac{1}{2}\rho g(\eta^2 - h^2)\Big)$$

3. To obtain the total energy, we integrate over the domain Ω :

$$\mathbb{E}(t) = \iint_{\Omega} E(\vec{x}, t)\, d\vec{x}$$

Total energy

Comparison between different ways of energy computation



Outline

Main conclusions

Preliminary results

- Different approaches for tsunami generation are not equivalent from energy viewpoint
  - \- : Passive generation overestimates tsunami energy
  - \+ : All lead to the equipartition between K and Π
- Energy propagation is nondispersive
- For long waves solving the additional energy equation is not equivalent to energy reconstruction a posteriori

Thank you for your attention !

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