On the dynamic generation of tsunamis by a moving bottom

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Towards more realistic dynamic source models

Physical assumptions

- Earth crust is a linear viscoelastic material
  - Kelvin-Voigt viscosity model
- Isotropic homogeneous or heterogeneous medium
- Fault modeled as a dislocation
  - **Dislocation** is a surface in continuous medium where the displacement field is discontinuous
  - The displacement field is increased by the amount of the Burgers vector along any loop enclosing the dislocation

$$\oint_C d\vec{u} = \vec{b}$$

  - Simplified situation with respect to fracture mechanics : location and displacement jump are known

Viscoelastodynamic governing equations

Strong formulation

$$
\begin{aligned}
\rho\frac{\partial^2\vec{u}}{\partial^2 t} - \nabla\cdot\underline{\underline{\sigma}} &= \vec{0}, && \vec{x}\in\Omega,\\
\underline{\underline{\sigma}}\cdot\vec{n} &= \vec{0}, && \vec{x}\in\Gamma_N,\\
\vec{u} &= \vec{0}, && \vec{x}\in\Gamma_D,\\
\vec{u}(\vec{x},0) = \vec{u}_t(\vec{x},0) &= \vec{0}, && \vec{x}\in\Omega,\\
\vec{u}^{\,+} - \vec{u}^{\,-} &= \vec{b}, && \vec{x}\in\Sigma \text{ (on the fault)}.
\end{aligned}
$$

Constitutive relations :

- Stress tensor is determined by a variant of Hooke's law

$$
\underline{\underline{\sigma}} = \lambda^*(\nabla\cdot\vec{u})\underline{\underline{I}} + 2\mu^*\underline{\underline{\varepsilon}}, \qquad \lambda^* = \lambda_r - i\lambda_i, \qquad \mu^* = \mu_r - i\mu_i,
$$

- Strain tensor $\underline{\underline{\varepsilon}} = \frac{1}{2}\left(\nabla\vec{u} + \nabla^t\vec{u}\right)$

Nonlinear shallow water equations

The most common model for tsunami waves

- Governing equations :

$$\begin{aligned} \frac{\partial \eta}{\partial t} + \nabla \cdot \left((h+\eta)\vec{v} \right) &= -\frac{\partial h}{\partial t}, \\ \frac{\partial \vec{v}}{\partial t} + \frac{1}{2}\nabla |\vec{v}|^2 + g\nabla \eta &= 0. \end{aligned}$$

η : free surface elevation with respect to still water level

$\vec{v}$: horizontal velocity

h : bathymetry

- Solved by finite volume scheme
- The coupling with FEM computation is done through bathymetry function $h = h(x, y, t)$

Coupling with hydrodynamics

General remarks

Active generation :
- We compute the first 10 seconds of the Earthquake and couple it with hydrodynamic solver
- For $t > 10s$ we assume that bottom remains static in its last configuration (at $t = 10s$)

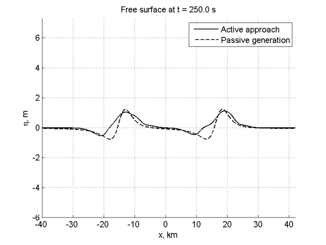
Passive generation :
- Static dislocation solution translated onto free surface as initial condition

Remark : This problem contains two different time scales : fast scale of seismic waves and slow scale of gravity waves.

Conclusions

Preliminary results

- Elevation wave has the right position
- Depression wave is slightly shifted due to larger extent of dynamic solution
- It seems that vertical bottom motion is followed by horizontal expansion of the deformation



- Passive generation can be suitable for warning systems
- If one wants to explain satelite tsunami records more sophisticated modeling is needed