Tsunami wave energy

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« Numerical methods for complex fluid flows »

Acknowledgements

Collaborators :

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Outline

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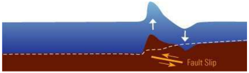
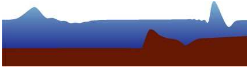
Physical context

Three stages of tsunami life

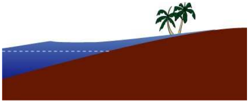
1. Generation



2. Propagation

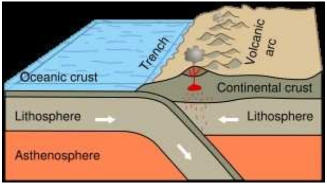


3. Inundation or run-up



津
波

- “tsu” is harbour,
- “nami” is wave



Energy of tsunamis

There is no common definition

Some references :

1. S. Tinti & E. Bortolucci. *Energy of Water Waves Induced by Submarine Landslides*. Pure Appl. Geophys. 157 (2000)
2. Z. Kowalik et al. *The Tsunami of 26 December 2004 : Numerical Modeling and Energy Considerations*. 22nd IUGG International Tsunami Symposium (2005)
3. T. Murty et al. *Leakage of the Indian Ocean Tsunami Energy into the Atlantic and Pacific Ocean*. J. Canadian Association of Exploration Geophysicists (2005)
4. A. Velichko et al. *Amplitude-energy characteristics of tsunami waves for various types of seismic sources generating them*. Physical Oceanography (2002)

Tsunami magnitude

Divergence in the definition

There exist at least seven different definitions :

Papadopoulos, Imamura (2001) : 12-point descriptive tsunami intensity scale. This scale is not related to any quantitative physical parameters.

Abe & Hatori (1985) : $M_t := a \lg h + b \log \Delta + c$

Kanamori (1972) : $M_w = \frac{2}{3}(\lg M_0 - 16.1)$

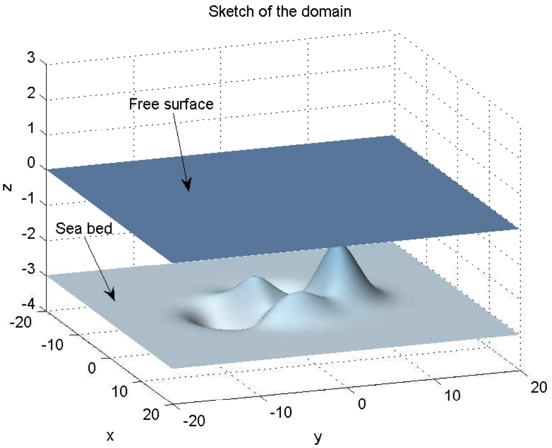
Murty & Loomis (1980) : $ML := 2(\lg E - 19)$

B. Levin, M. Nosov. *Physics of tsunamis*, Springer, 2009

...The definition of magnitude based on the wave energy is, naturally, the most adequate definition, from a physical point of view. However, it is not always possible to calculate the wave energy...

Outline

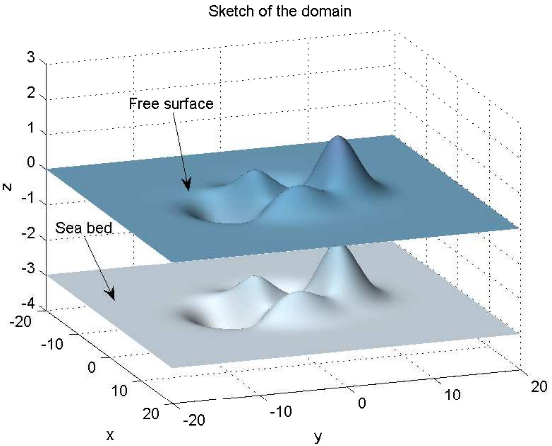
Put coseismic displacements directly on the free surface and let it propagate :



Traditional approach

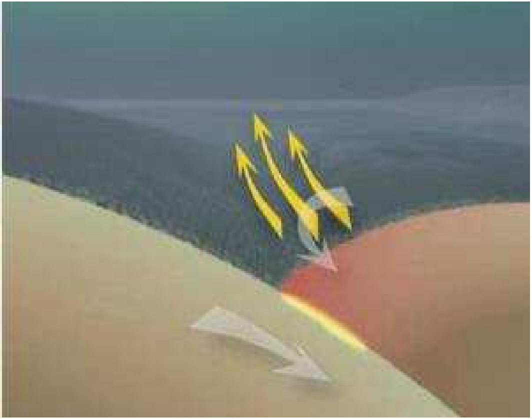
Approaches to generation

Put coseismic displacements directly on the free surface and let it propagate :



Tsunami generation process

Video illustration by courtesy of J. Sharpe



Tsunami generation modelling

Different scenarios of bottom motion

We consider the following situations :

1. Passive generation
   - $\Rightarrow$ Initial Value Problem (IVP)
2. Instantaneous seabed deformation

$$h(\vec{x}, t) = h_0(\vec{x}) + \mathcal{H}(t)\zeta(\vec{x})$$

3. Exponential scenario

$$h(\vec{x}, t) = h_0(\vec{x}) + (1 - e^{-\alpha t})\zeta(\vec{x})$$

Reference :

D. Dutykh & F. Dias. *Water waves generated by a moving bottom*. A. Kundu (ed). Tsunami and nonlinear waves, 2007

Analytical solutions

Linearized water waves : Cauchy-Poisson problem

- Free surface elevation

$$\eta(\vec{x},t) = \frac{1}{(2\pi)^2} \iint_{\mathbb{R}^2} \frac{\widehat{\zeta}(\vec{k}) e^{i\vec{k}\cdot\vec{x}}}{\cosh(|\vec{k}|h)} \frac{1}{2\pi i} \int_{\mu-i\infty}^{\mu+i\infty} \frac{s^2 \mathrm{T}(s)}{s^2+\omega^2} e^{st} ds\, d\vec{k}$$

- Velocity potential

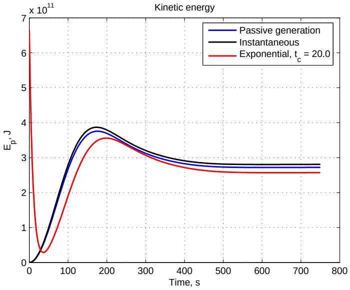
$$\hat{\phi}(\vec{x},z,t) = \frac{gs\widehat{\zeta}(\vec{k})\mathrm{T}(s)}{\cosh(|\vec{k}|h)(s^2+\omega^2)} \left(\frac{s^2}{g|\vec{k}|} \sinh(|\vec{k}|z) - \cosh(|\vec{k}|z) \right)$$

- Compute the energies

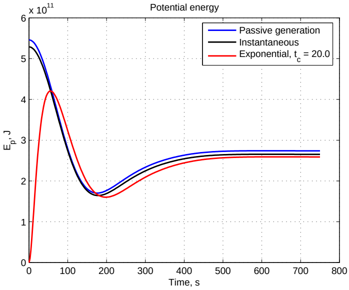
$$K = \frac{1}{2} \int_{-h}^{\eta} \iint_{\mathbb{R}^2} |\nabla \phi|^2 \, d\vec{x} dz, \quad \Pi = \frac{g}{2} \iint_{\mathbb{R}^2} \eta^2 \, d\vec{x}$$

Energy analysis of generation process - I

Kinetic and potential energies



(a) Kinetic energy



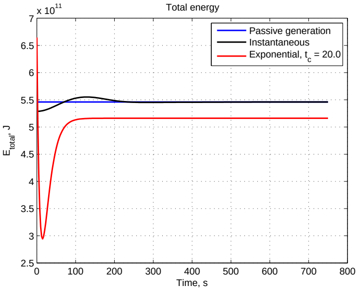
(b) Potential energy

Reference :

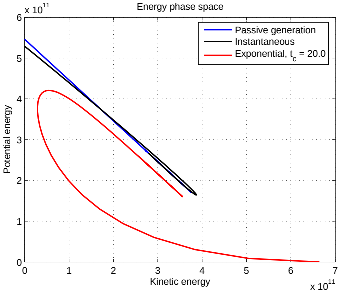
D. Dutykh & F. Dias. *Energy of tsunami waves generated by bottom motion*. Proc. R. Soc. A., **465**, 725–744, 2009

Energy analysis of generation process - II

Total energy and phase space



(c) Total energy



(d) Energy phase space

Reference :

D. Dutykh & F. Dias. *Energy of tsunami waves generated by bottom motion*. Proc. R. Soc. A., **465**, 725–744, 2009

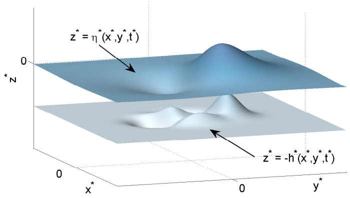
Outline

Energy equation derivation

Departure point : incompressible Euler equations

- Incompressibility condition

$$\nabla \cdot \vec{u} = 0$$



- Momentum balance

$$\frac{\partial \vec{u}}{\partial t} + \nabla \cdot \left(\vec{u} \otimes \vec{u} + \frac{p}{\rho} \mathbb{I} \right) = \vec{g}$$

- Energy equation

$$\frac{\partial e}{\partial t} + \nabla \cdot \left[\left(e + \frac{p}{\rho} \right) \vec{u} \right] = 0, \qquad e := K + \Pi = \frac{1}{2} |\vec{u}|^2 + gz$$

Energy equation derivation for long waves

Integration over the depth

After integrating energy equation over the depth :

$$E_t + \nabla \cdot \vec{\Phi} + P = 0$$

- Total energy

$$E = \int_{-h}^{\eta} \Big(\frac{1}{2}\rho(|\vec{u}|^2 + w^2) + \rho g z \Big) dz$$

- Energy flux

$$\vec{\Phi} = \int_{-h}^{\eta} \Big(\frac{1}{2}\rho(|\vec{u}|^2 + w^2) + p + \rho g z \Big) \vec{u}\, dz$$

- Energy input term

$$P = p_s \eta_t + p_b h_t$$

After integrating energy equation over the depth :

$$E_t + \varepsilon \nabla \cdot \vec{\Phi} + \varepsilon P = 0$$

- Total energy

$$E = \int_{-h}^{\varepsilon\eta} \Big(\frac{\varepsilon}{2} (\varepsilon |\vec{u}|^2 + \frac{\varepsilon}{\mu^2} w^2) + z \Big) dz$$

- Energy flux

$$\vec{\Phi} = \int_{-h}^{\varepsilon\eta} \Big(\frac{\varepsilon}{2} (\varepsilon |\vec{u}|^2 + \frac{\varepsilon}{\mu^2} w^2) + \varepsilon p + z \Big) \vec{u} \, dz$$

- Energy input term

$$P = \varepsilon p_b \zeta_t$$

NSW2E : Nonlinear Shallow Water Equations with Energy

- Mass conservation

$$H_t + \nabla \cdot (H\vec{u}) = 0$$

- Momentum conservation

$$(H\vec{u})_t + \nabla \cdot \left(H\vec{u} \otimes \vec{u} + \frac{g}{2}H^2\right) = gH\nabla h$$

- Energy equation

$$E_t + \nabla \cdot \left(\left(E + \frac{1}{2}\rho g H^2\right)\vec{u}\right) = \rho g H \zeta_t$$

Governing equations - II

Boussinesq equations with energy

- Mass conservation

$$H_t + \nabla \cdot (H\vec{u}) = 0$$

- Momentum conservation

$$(H\vec{u})_t + \nabla \cdot \big(H\vec{u} \otimes \vec{u} + \frac{g}{2}H^2\big) + \\ \Big(\frac{h^3}{6}\nabla\big(\nabla \cdot (\frac{H\vec{u}}{d})\big) - \frac{h^2}{2}\nabla\big(\nabla \cdot (H\vec{u})\big)\Big)_t = gH\nabla h$$

- Energy equation

$$E_t + \nabla \cdot \Big(\big(E + \frac{1}{2}\rho g H^2\big)\vec{u}\Big) + \mathcal{O}(\varepsilon\mu^2) = \rho g H \zeta_t$$

Energy equation modification

How to extract the wave energy ?

Total energy equation :

$$E_t + \nabla \cdot \Big(\big(E + \frac{1}{2}\rho g H^2\big)\vec{u} \Big) = \rho g H \zeta_t$$

- kinetic wave energy
- potential wave energy
- potential energy of the water column

Define the wave energy :

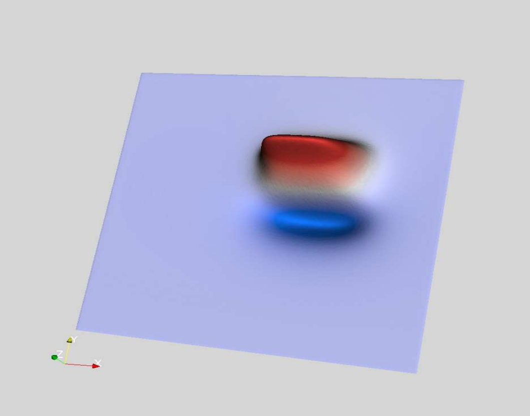
$$E_w := E + \frac{1}{2}\rho g h(\vec{x}, t)^2$$

Wave energy equation :

$$E_{wt} + \nabla \cdot \Big(\big(E_w + \frac{1}{2}\rho g H^2 - \frac{1}{2}\rho g h^2\big)\vec{u} \Big) = \rho g (H - h)\zeta_t$$

Active generation of a tsunami wave - I

Free surface elevation by VOLNA code



Reference :

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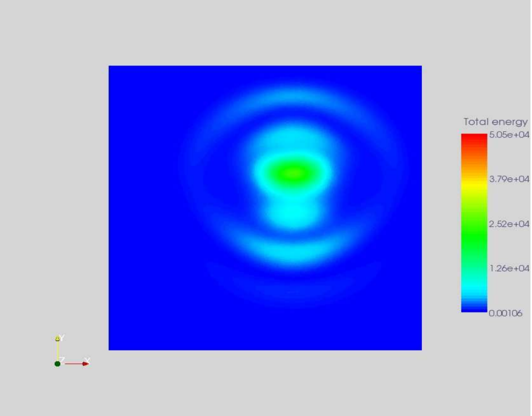
Active generation of a tsunami wave - II

Total wave energy density computation by VOLNA code



Reference :

D. Dutykh & F. Dias. *Energy of tsunami waves generated by bottom motion*. Proc. R. Soc. A., **465**, 725–744, 2009

Energy reconstruction

Computation of the total energy from NSWE solutions

Why to add a supplementary energy equation ?

1. We solve NSWE $\Rightarrow (h, H, \vec{u})$
2. Using this information, we approximate the energy density :

$$E \approx \frac{1}{2}\rho H|\vec{u}|^2 + \frac{1}{2}\rho g\eta^2$$

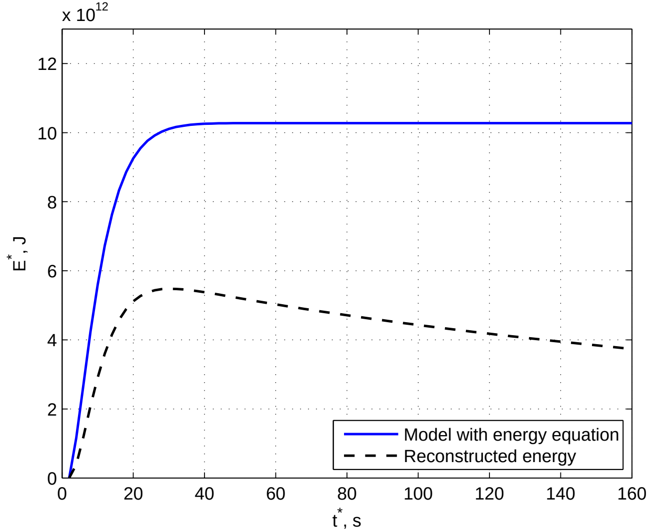
3. To obtain the total **wave** energy, we integrate over the domain Ω :

$$\mathbb{E}(t) = \iint\limits_{\Omega} E(\vec{x}, t)d\vec{x}$$

Let us compare two methods. . .

Total energy

Comparison between two ways of energy computation



Outline

Conclusions

Conclusions :

- Generation process was analyzed from energy point of view
- Long wave energy equation was derived
- Simulations involving energy were presented

Perspectives :

- Adaptation to landslide generated tsunamis
- Connection between wave energy and extreme run-up...

Thank you for your attention !

`http://www.lama.univ-savoie.fr/~dutykh`