Simulation of free surface compressible flows via a two fluid model

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Outline

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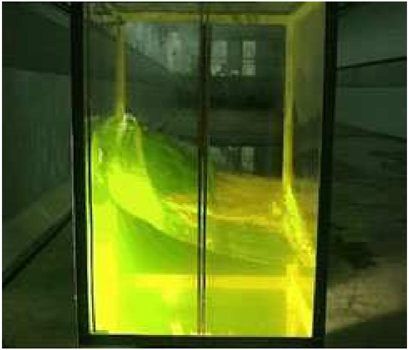
1. Physical context
   - Main applications
   - Experimental results
2. Mathematical model
   - Governing equations
   - Equation of state
   - Formal limit
   - Numerical scheme
3. Simulation results
4. Perspectives

Physical phenomena - I

Two applications which motivated this study

- Wave sloshing in liquefied natural gas (LNG) carriers



Physical phenomena - II

Two applications which motivated this study

- Wave impact on walls and coastal structures

FIG.: Experiments at GWK (Hannover)

Wave impacts on a wall

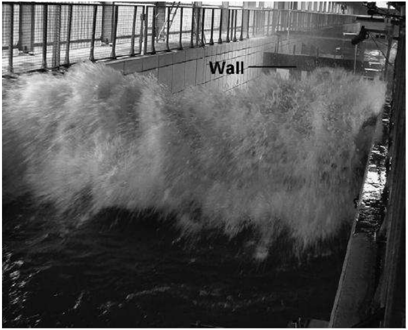
Ref : Bullock, Obhrai, Peregrine, Bredmose, 2007

Challenge :

Determine efforts exerted by waves on structures

Impacts classification :

- low-aeration : the water adjacent to the wall contains typically 5% of air
- high-aeration : higher level of entrained air with clear evidence of entrapment



Main results of the experimental study

Ref : Bullock, Obhrai, Peregrine, Bredmose, 2007

- Low-aeration impact
  - temporary and spatially localised pressure impulse
- High-aeration impact
  - less localised pressure spike with a longer rise time, fall time and duration
  - peak values of the pressure are lower

Conclusion :

« . . .Even when the pressures during a high-aeration impact are lower, the fact that the impact is less spatially localised and lasts longer may well lead to a higher total impulse. . . »

Influence of aeration

Ideas for mathematical modelling

For low-aeration water wave impact ($\alpha_g \approx 0.05$) :

- Sound speed drops down to $\approx 54 \frac{m}{s}$
- Compressible effects are very important
    - Mach number is not tiny anymore
- CFL condition is not so severe
    - Explicit in time scheme

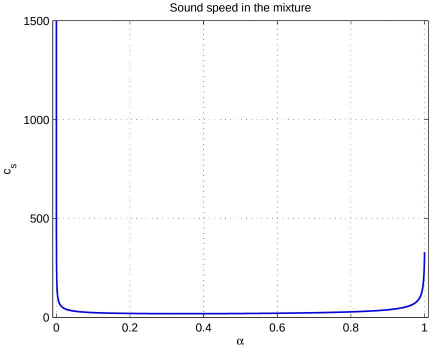


FIG.: Sound speed in the air/water mixture

Outline

Mass conservation for each phase :

$$\partial_t(\alpha^\pm \rho^\pm) + \nabla \cdot (\alpha^\pm \rho^\pm \vec{u}) = 0,$$

Momentum equation :

$$\partial_t(\rho\vec{u}) + \nabla \cdot \big(\rho\vec{u} \otimes \vec{u} + p\mathbb{I}\big) = \rho\vec{g},$$

Energy conservation :

$$\partial_t\big(\rho E\big) + \nabla \cdot \big(\rho H\vec{u}\big) = \rho\vec{g} \cdot \vec{u},$$

$$\alpha^+ + \alpha^- = 1,\ \rho := \alpha^+\rho^+ + \alpha^-\rho^-,\ H := E + \frac{p}{\rho},\ E := e + \frac{1}{2}|\vec{u}|^2.$$

Two-phase homogenous model - II

Equation of state

- Ideal gas law for light fluid :

$$\begin{aligned} p^- &= (\gamma - 1)\rho^- e^-, \\ e^- &= c_v^- T^-, \end{aligned}$$

- Tait's law for heavy fluid :

$$\begin{aligned} p^+ + \pi_0 &= (\mathcal{N} - 1)\rho^+ e^+, \\ e^+ &= c_v^+ T^+ + \frac{\pi_0}{\mathcal{N}\rho^+}, \end{aligned}$$

where γ, $c_v^\pm$, π_0, $\mathcal{N}$ are constants

Additional assumption : Two phases are in thermodynamic equilibrium :

$$p := p^+ = p^-, \qquad T := T^+ = T^-$$

Motivation for the choice of this model

Trade-off between model complexity and accuracy of the results

Main reasons

- There is **no special treatment** of the inerface
- Can naturally handle **wave breaking**
- Compressible and hyperbolic
- Computations are **not expensive**

We believe that this model gives qualitatively correct results for the flow and right impact pressure

Limit to a discontinous two-phase system

Two phases separated by an interface

- One can derive the following equation for $\alpha^{\pm}$:

$$\alpha_t^{\pm} + (\vec{u} \cdot \nabla)\alpha^{\pm} + \alpha^{+}\alpha^{-}\frac{\delta^{\pm}\gamma}{\bar{\gamma}}\nabla \cdot \vec{u} = 0 \qquad (1)$$

- We separate the phases : $\alpha^{-} = H(z - \eta(\vec{x}, t))$
  - It follows that : $\alpha^{+}\alpha^{-} = 0$

- Substituting in (1) gives

$$\eta_t + (u, v) \cdot \nabla_x \eta = w$$

- Governing equations become

$$\begin{aligned} \rho_t + (\vec{u} \cdot \nabla)\rho + \rho\nabla \cdot \vec{u} &= 0, \\ \vec{u}_t + (\vec{u} \cdot \nabla)\vec{u} + \frac{\nabla p}{\rho} &= \vec{g}. \end{aligned}$$

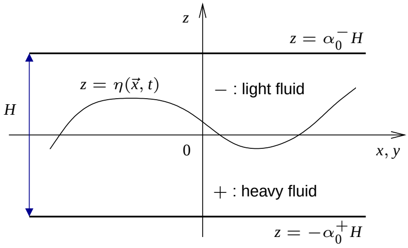


FIG.: Sketch of the flow.

System of balance laws

Rewrite governing equations :

$$\frac{\partial w}{\partial t} + \nabla \cdot \mathcal{F}(w) = \mathcal{S}(\vec{x}, t, w),$$

Integrate them over control volume :

$$\frac{d}{dt}\int_K w \, d\Omega + \int_{\partial K} \mathcal{F}(w) \cdot \vec{n}_{KL} \, d\sigma = \int_K \mathcal{S}(w) \, d\Omega$$

Introduce cell averages :

$$w_K(t) := \frac{1}{\mathrm{vol}(K)} \int_K w(\vec{x}, t) \, d\Omega$$

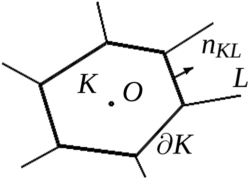


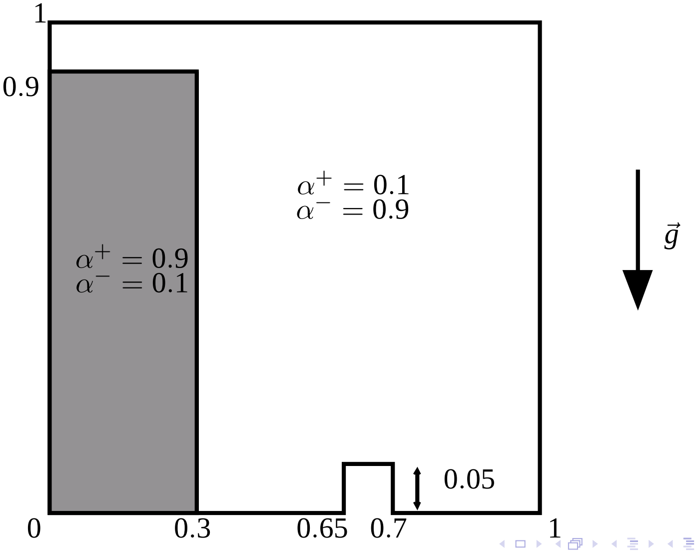
FIG.: Control volume

How to express $(\mathcal{F} \cdot \vec{n})|_{\partial K}$ in terms of $\{w_K\}_{K \in \Omega}$?

Outline

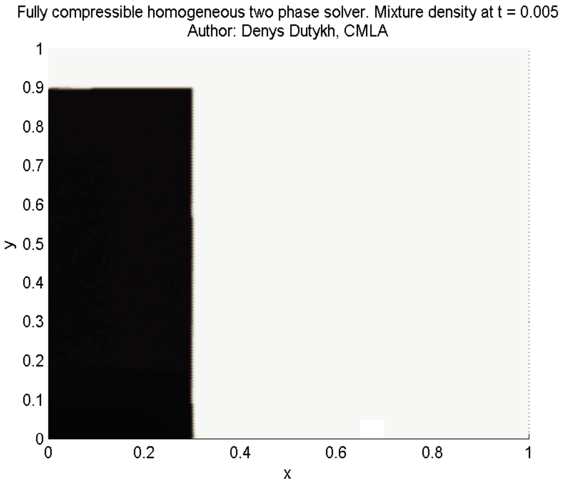
Water column test case - I

Geometry and description of the test case



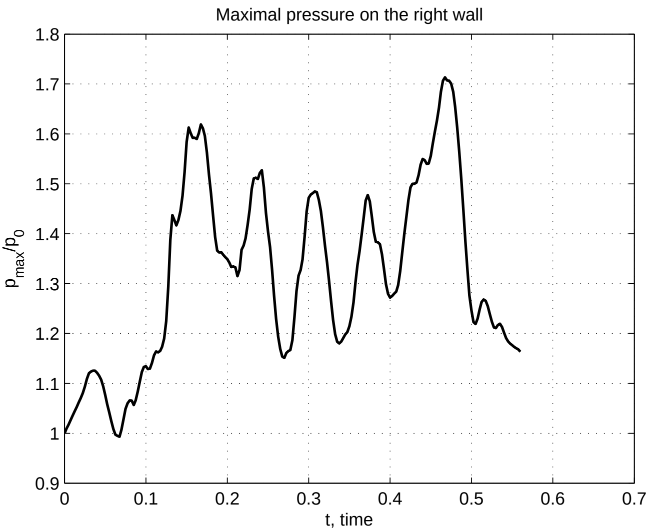
Water column test case - II

Gravity acceleration $g = 100m/s^2$,in heavy fluid $\alpha^+ = 0.9$, in light fluid $\alpha^+ = 0.1$



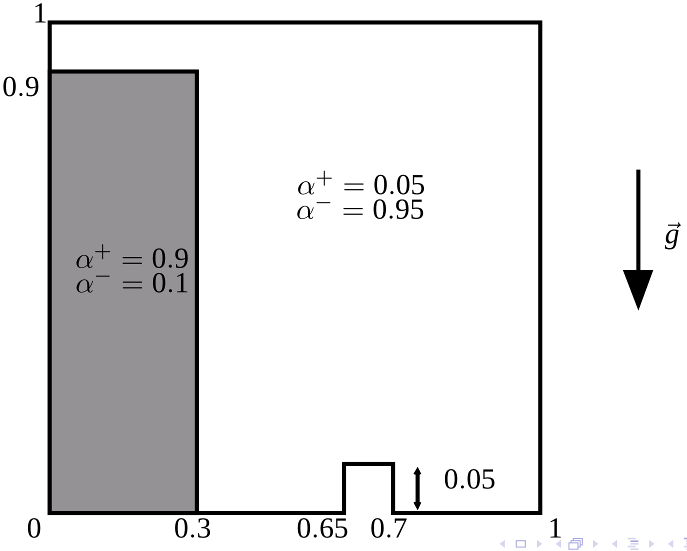
Maximal pressure on the right wall

as a function of time $t \longmapsto \max_{(x,y)\in 1\times[0,1]} p(x,y,t)$



Water column test case - III

Lighter gas case



Maximal pressure on the right wall

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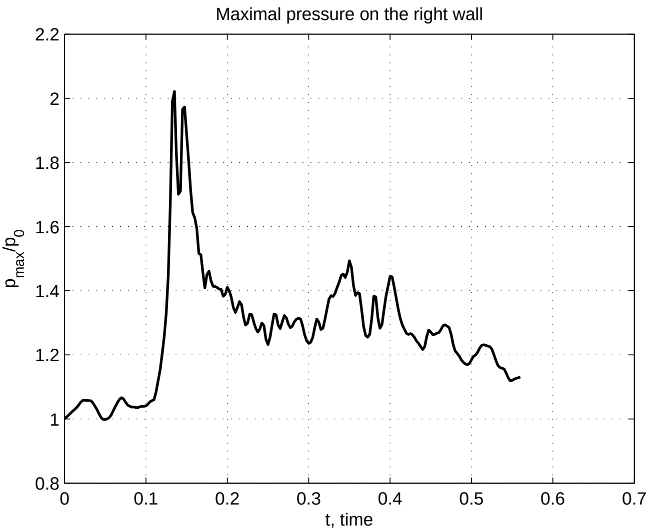
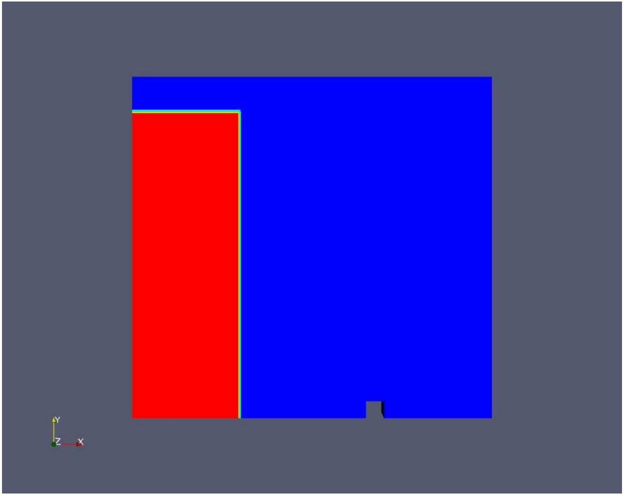


Incompressible computation : volume fraction

Two-fluids Navier-Stokes equations solver : OpenFOAM



Impact pressures at the right wall

Comparison between compressible and incompressible models

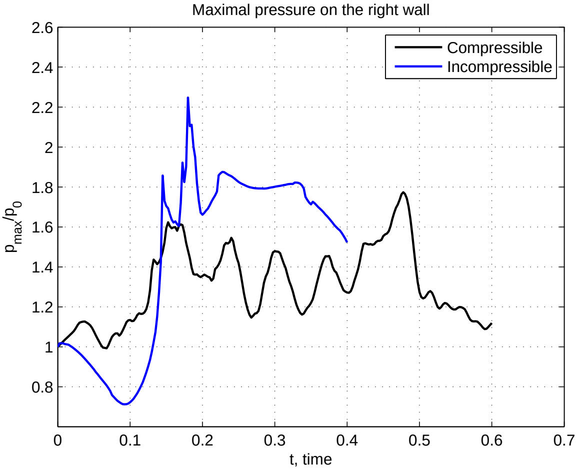


FIG.: Gas volume fraction $\alpha_g = 0.9$

Impact pressures at the right wall

Comparison between compressible and incompressible models

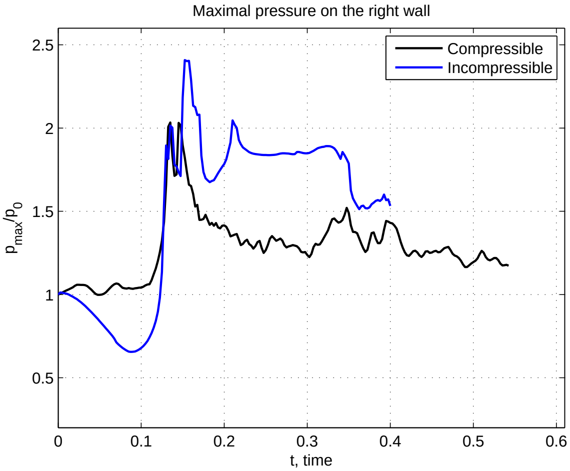


FIG.: Gas volume fraction $\alpha_g = 0.95$

Outline

Physics :

- Quantitative comparison with 6-equations model
- Parametric study (aeration and wave breaking influence on impact pressures)

Mathematics :

- Formal justification of 4-equations model

Numerics :

- Towards pure phases computations
  - Implicit time stepping
  - Low Mach number problem

Thank you for your attention !

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