Simulation of free surface compressible motions via a two fluid model

Denys Dutykh[1] Frédéric Dias[1] Jean-Michel Ghidaglia[1]

[1]Ecole Normale Supérieure de Cachan,
Centre de Mathématiques et de Leurs Applications,
LRC Méso CMLA/CEA DAM IdF

《 GDR MOAD: session Lyon 2008 》

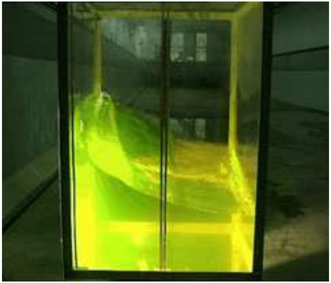
Contents

Contents

Physical phenomena

Two applications which motivated this study

- Wave sloshing in Liquefied Natural Gas (LNG) carriers



- Wave impacts on coastal structures

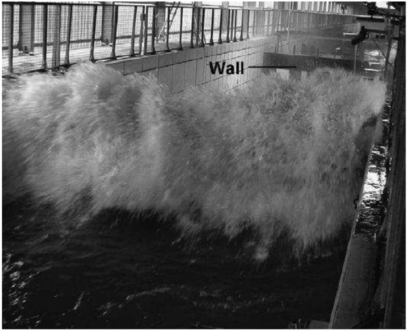
FIG.: GWK, Hannover

Wave impacts on a wall

Ref : Bullock, Obhrai, Peregrine, Bredmose, 2007

Impacts classification :

- low-aeration : the water adjacent to the wall contains typically 5% of air
- high-aeration : higher level of entrained air with clear evidence of entrapment



Main results of the experimental study

Ref : Bullock, Obhrai, Peregrine, Bredmose, 2007

- Low-aeration impact
  - temporary and spatially localised pressure impulse
- High-aeration impact
  - less localised pressure spike with a longer rise time, fall time and duration
  - peak values of the pressure are lower

Conclusion :

« Even when the pressures during a high-aeration impact are lower, the fact that the impact is less spatially localised and lasts longer may well lead to a higher total impulse »

Influence of aeration

Ideas for mathematical modelling

For low-aeration water wave impact ($\alpha_g \approx 0.05$) :

- Sound speed drops down to $\approx 54 \frac{m}{s}$
- Compressible effects are very important
  - Mach number is not tiny anymore
- CFL condition is not so severe
  - Explicit in time scheme

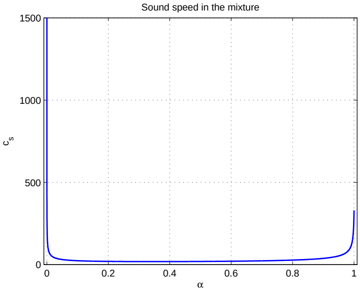


FIG.: Sound speed in the air/water mixture

Contents

Two-phase homogenous model - I

Governing equations

Mass conservation for each phase :

$$\partial_t(\alpha^\pm \rho^\pm) + \nabla \cdot (\alpha^\pm \rho^\pm \vec{u}) = 0,$$

Momentum equation :

$$\partial_t(\rho \vec{u}) + \nabla \cdot \big(\rho \vec{u} \otimes \vec{u} + p\mathbb{I}\big) = \rho \vec{g},$$

Energy conservation :

$$\partial_t\big(\rho E\big) + \nabla \cdot \big(\rho H \vec{u}\big) = \rho \vec{g} \cdot \vec{u},$$

$$\alpha^+ + \alpha^- = 1, \; \rho := \alpha^+\rho^+ + \alpha^-\rho^-, \; H := E + \frac{p}{\rho}, \; E := e + \frac{1}{2}|\vec{u}|^2.$$

Two-phase homogenous model - II

Equation of state

- Ideal gas law for light fluid :

$$\begin{aligned} p^- &= (\gamma - 1)\rho^- e^-, \\ e^- &= c_v^- T^-, \end{aligned}$$

- Tait's law for heavy fluid :

$$\begin{aligned} p^+ + \pi_0 &= (\mathcal{N} - 1)\rho^+ e^+, \\ e^+ &= c_v^+ T^+ + \frac{\pi_0}{\mathcal{N}\rho^+}, \end{aligned}$$

where γ, $c_v^\pm$, π_0, $\mathcal{N}$ are constants

Additional assumption : Two phases are in thermodynamic equilibrium :

$$p := p^+ = p^-, \qquad T := T^+ = T^-$$

Motivation for the choice of this model

Trade-off between model complexity and accuracy of the results

Main reasons

- This model is **hyperbolic**
- There is **no special treatment** of the free surface
- We have only **four** equations in 1D
- Equations do not contain **nonconservative products**
- Eigenvalues and eigenvectors can be computed **analytically**

$\Rightarrow$ computation is not expensive

We believe that this model gives qualitatively correct results for the flow and right impact pressure

Limit to the classical free surface model

For illustration we take the barotropic case

- Consider the following system :

$$\begin{aligned} \partial_t(\alpha^\pm \rho^\pm) + \nabla \cdot (\alpha^\pm \rho^\pm \vec{u}) &= 0, \\ \partial_t(\rho \vec{u}) + \nabla \cdot (\rho \vec{u} \otimes \vec{u}) + \nabla p &= \rho \vec{g}, \end{aligned}$$

- Equation of state : $p = p(\rho^+) = p(\rho^-)$
- Speeds of sound are given by $(c^\pm)^2 := \frac{\partial p}{\partial \rho^\pm}$
- We define the following quantities :

$$\gamma^\pm := \frac{\rho^\pm (c^\pm)^2}{p}, \qquad \bar{\gamma} := \alpha^+ \gamma^- + \alpha^- \gamma^+, \qquad \delta^\pm \gamma = \gamma^\pm - \gamma^\mp$$

Limit to a discontinous two-phase system

Two phases separated by an interface

- One can derive the following equation for $\alpha^{\pm}$:

$$\alpha_t^{\pm} + (\vec{u}\cdot\nabla)\alpha^{\pm} + \alpha^{+}\alpha^{-}\frac{\delta^{\pm}\gamma}{\bar{\gamma}}\nabla\cdot\vec{u} = 0 \qquad (1)$$

- We separate the phases : $\alpha^{-} = H(z - \eta(\vec{x}, t))$
  - It follows that : $\alpha^{+}\alpha^{-} = 0$

- Substituting in (1) gives

$$\eta_t + (u, v)\cdot\nabla_x\eta = w$$

- Governing equations become

$$\begin{aligned} \rho_t + (\vec{u}\cdot\nabla)\rho + \rho\nabla\cdot\vec{u} &= 0, \\ \vec{u}_t + (\vec{u}\cdot\nabla)\vec{u} + \frac{\nabla p}{\rho} &= \vec{g}. \end{aligned}$$

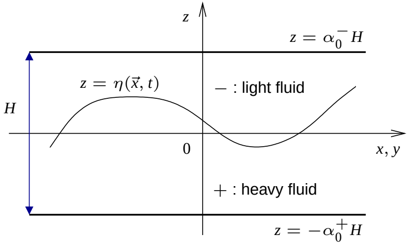


FIG.: Sketch of the flow.

Contents

System of balance laws

Rewrite governing equations :

$$\frac{\partial w}{\partial t} + \nabla \cdot \mathcal{F}(w) = \mathcal{S}(\vec{x}, t, w),$$

Integrate them over control volume :

$$\frac{d}{dt} \int_K w \; d\Omega + \int_{\partial K} \mathcal{F}(w) \cdot \vec{n}_{KL} \; d\sigma = \int_K S(w) \; d\Omega$$

Introduce cell averages :

$$w_K(t) := \frac{1}{\mathrm{vol}(K)} \int_K w(\vec{x}, t) \; d\Omega$$

How to express $(\mathcal{F} \cdot \vec{n})|_{\partial K}$ in terms of $\{w_K\}_{K \in \Omega}$?

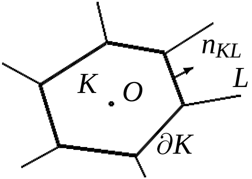


FIG.: Control volume

Finite volumes scheme

Volumes Finis à Flux Caractéristique (VFFC)

Use numerical flux of VFFC scheme to discretize advection operator :

$$\Phi(w_K, w_L; \vec{n}_{KL}) = \frac{\mathcal{F}_n(w_K) + \mathcal{F}_n(w_L)}{2} - U(\mu; \vec{n}_{KL})\frac{\mathcal{F}_n(w_L) - \mathcal{F}_n(w_K)}{2}$$

where μ is a mean state

$$\mu := \frac{\mathrm{vol}(K)w_K + \mathrm{vol}(L)w_L}{\mathrm{vol}(K) + \mathrm{vol}(L)}$$

and $U(\mu; \vec{n}_{KL})$ is the sign matrix

$$U := \mathrm{sign}(\mathbb{A}_n) \equiv \mathbf{R}\,\mathrm{sign}(\Lambda)\mathbf{R}^{-1}, \qquad \mathbb{A}_n := \frac{\partial(\mathcal{F}\cdot\vec{n})(w)}{\partial w}$$

Remark : Since, the advection operator is relatively simple, U can be computed analytically.

Second-order sequel : MUSCL scheme

Monotone Upstream-centered Schemes for Conservation Laws

- We find our solution in class of affine by cell functions :

$$w_K(\vec{x},t) := \bar{w}_K + (\nabla w)_K(\vec{x} - \vec{x}_0)$$

  - x_0 is the barycenter of the cell K
- Gradient reconstruction :
  - Least-squares method
  - Green-Gauss reconstruction
- Slope limiter :
  - Barth-Jespersen (1989)
  - no limiter at all

Time integration : SSP-RK3(4) scheme

Ref : Spiteri & Ruuth (2002), SIAM J. Numer. Anal.

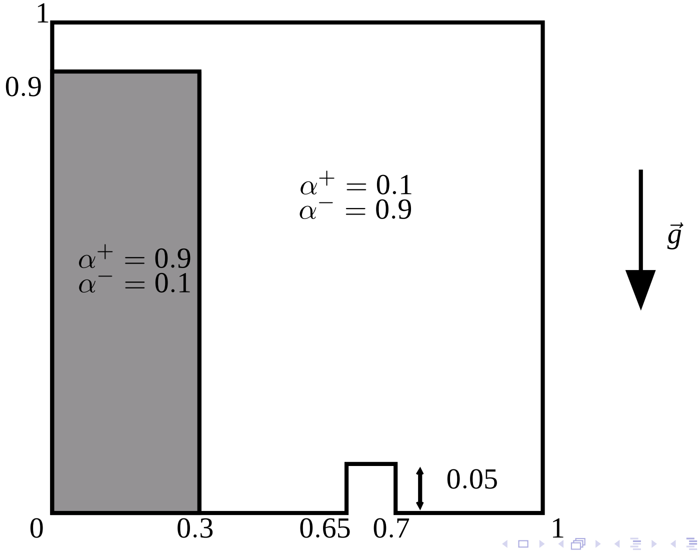
Third order 4-stage scheme with $CFL = 2$:

$$
\begin{aligned}
u^{(1)} &= u^{(n)} + \frac{1}{2}\Delta t L(u^{(n)}), \\
u^{(2)} &= u^{(1)} + \frac{1}{2}\Delta t L(u^{(1)}), \\
u^{(3)} &= \frac{2}{3}u^{(n)} + \frac{1}{3}u^{(2)} + \frac{1}{6}\Delta t L(u^{(n)}), \\
u^{(n+1)} &= u^{(3)} + \frac{1}{2}\Delta t L(u^{(3)}),
\end{aligned}
$$

Contents

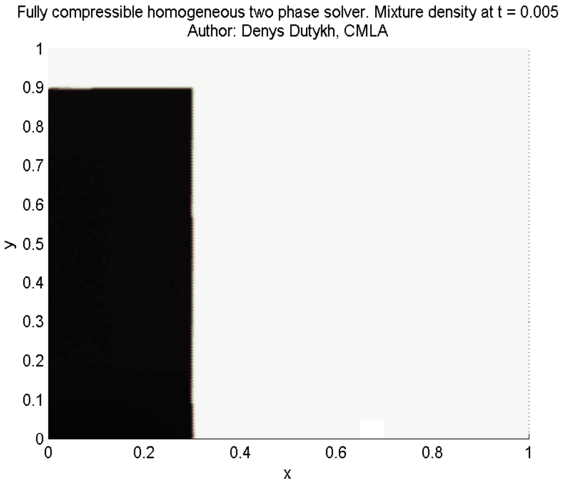
Water column test case - I

Geometry and description of the test case



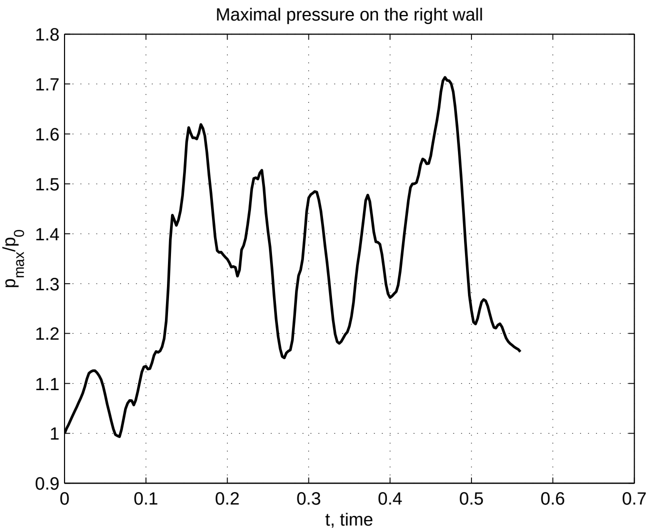
Water column test case - II

Gravity acceleration $g = 100 m/s^2$,in heavy fluid $\alpha^+ = 0.9$, in light fluid $\alpha^+ = 0.1$



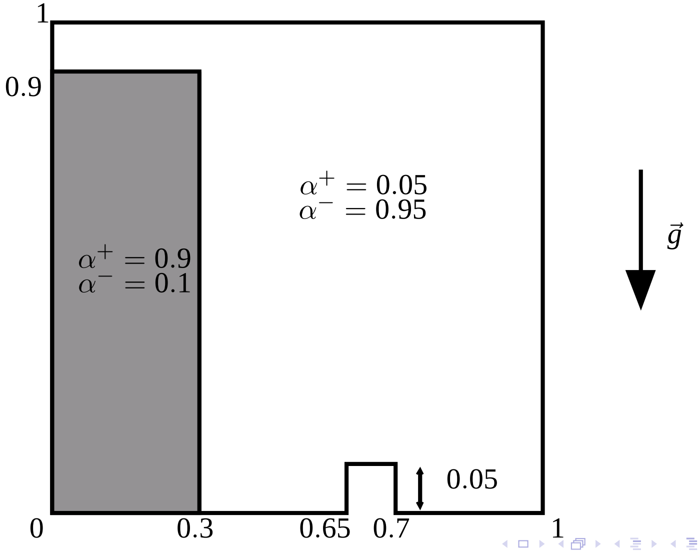
Maximal pressure on the right wall

as a function of time $t \longmapsto \max_{(x,y)\in 1\times[0,1]} p(x,y,t)$



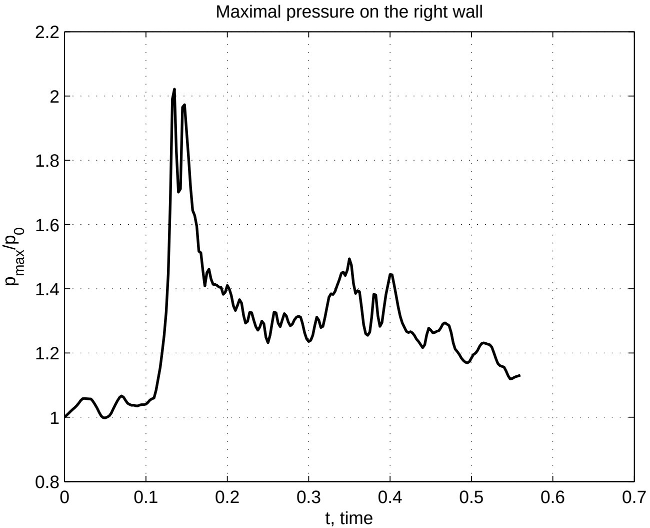
Water column test case - III

Lighter gas case



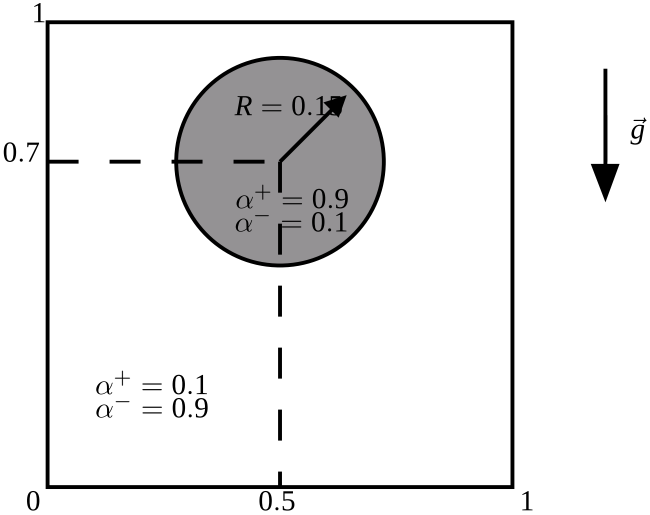
Maximal pressure on the right wall

as a function of time $t \longmapsto \max_{(x,y)\in 1\times[0,1]} p(x,y,t)$



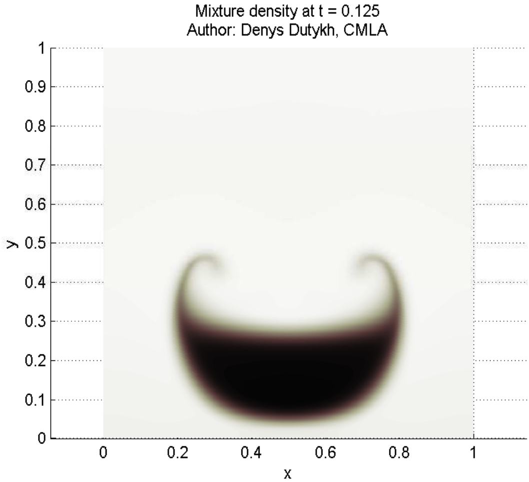
Water drop test case - I

Geometry and description of the test case



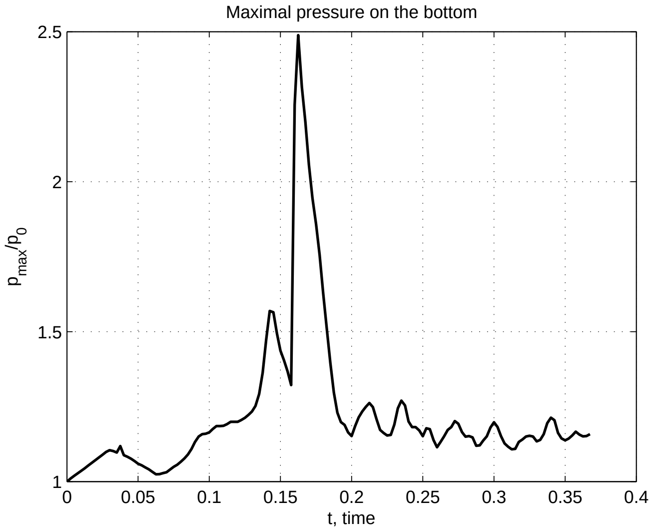
Water drop test case - II

Gravity acceleration $g = 100m/s^2$



Maximal bottom pressure

as a function of time $t \longmapsto \max_{(x,y)\in[0,1]\times 0} p(x,y,t)$



Contents

Perspectives

Directions for future work

Physics :

- Quantitative comparison with 6-equations model
- Parametric study : aeration influence on impact pressures
- Wave breaking influence

Mathematics :

- Formal justification of 4-equations model

Numerics :

- Towards pure phases computations
  - Implicit time stepping
  - Low Mach number problem

Thank you for your attention !

http://www.cmla.ens-cachan.fr/~dutykh