Dispersive wave runup and some related amplification phenomena

Denys Dutykh[1]

Chargé de Recherche CNRS

[1]LAMA, Université de Savoie
73376 Le Bourget-du-Lac, France

« MIT-2011 Conference »

Acknowledgements

To my collaborators

- Dimitrios Mitsotakis: Institute for Mathematics and its Applications, University of Minnesota
- Theodoros Katsaounis: Assistant Professor, University of Crete
- Frédéric Dias: Professor, University College Dublin (on leave from ENS de Cachan)
- Themistoklis Stefanakis: PhD student, UCD & ENS de Cachan

Wave runup

Definition of the wave runup

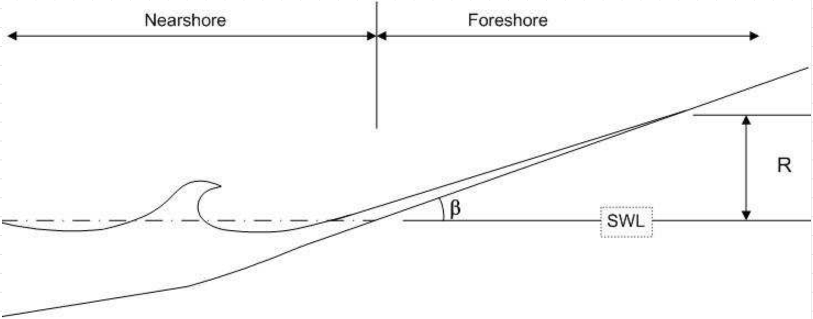


Figure: SWL indicates « Still Water Level »

Definition (Sorensen (1997), [Sor97]):

Wave runup is the maximum vertical extent of wave uprush on a beach or a structure above the still water level

The modified Peregrine system

Conservative sequel to the Peregrine system [Per67]:

$$H_t + Q_x = 0$$

$$\Big(\big(1+\frac{1}{3}H_x^2-\frac{1}{6}HH_{xx}\big)Q-\frac{1}{3}H^2Q_{xx}-\frac{1}{3}HH_xQ_x\Big)_t+\Big(\frac{Q^2}{H}+\frac{g}{2}H^2\Big)_x = gHh_x$$

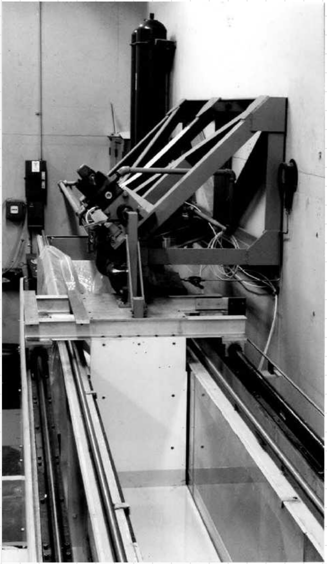
- FV for the advection, FD for the dispersion
- Well-balanced hydrostatic reconstruction
- Explicit SSP-RK3 time discretization

For more details see [DKM11]:

D. Dutykh, T. Katsaounis, D. Mitsotakis. *Finite volume schemes for dispersive wave propagation and runup*. J. Comput. Phys., **230**, 3035–3061, 2011

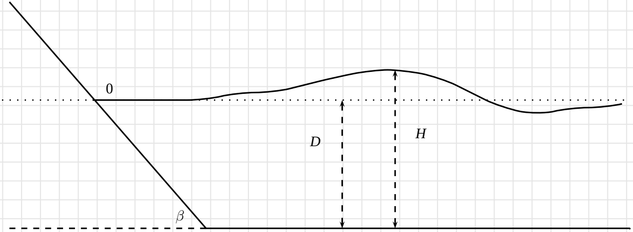
W.M. Keck Laboratory of Hydraulics, Caltech

PhD thesis of Costas Synolakis



Solitary wave runup on a plane beach

Experiments by C. Synolakis [Syn87], Caltech



Bottom shape:

$$-h(x) = \begin{cases} -x \tan\beta, & x \le \cot\beta, \\ -1, & x > \cot\beta, \end{cases}$$

Initial condition:

$$\eta_0(x) = A_s \mathrm{sech}^2\left(\lambda(x - X_0)\right), \quad u_0(x) = -c_s \frac{\eta_0(x)}{D_0 + \eta_0(x)}$$

Solitary wave runup: $A_s = 0.04$

Data from C. Synolakis (1987), $\beta = 2.88^\circ$

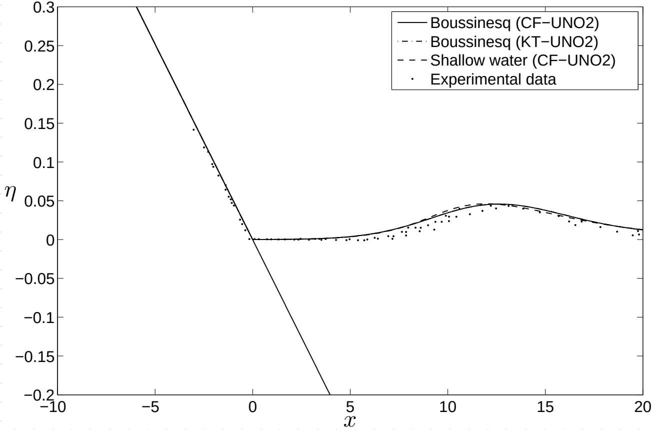


Figure: $t = 20$ s

Solitary wave runup: $A_s = 0.04$

Data from C. Synolakis (1987), $\beta = 2.88^\circ$

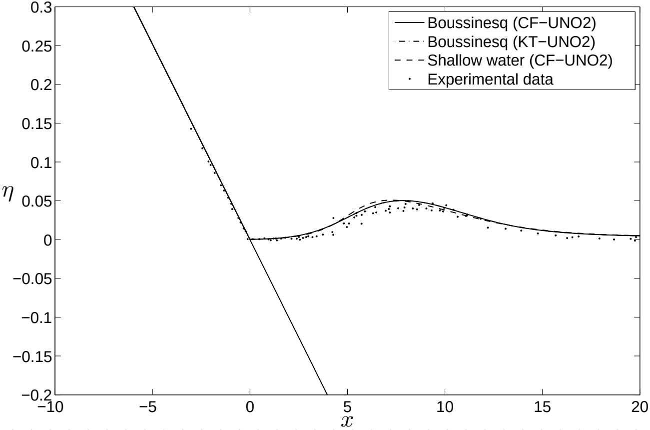


Figure: $t = 26$ s

Solitary wave runup: $A_s = 0.04$

Data from C. Synolakis (1987), $\beta = 2.88^\circ$

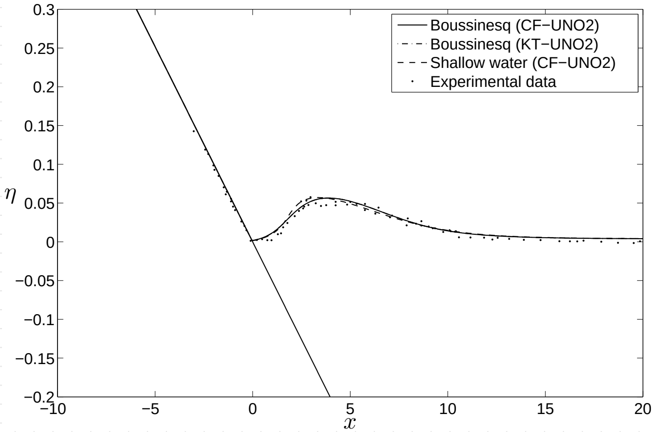


Figure: $t = 32$ s

Solitary wave runup: $A_s = 0.04$

Data from C. Synolakis (1987), $\beta = 2.88^{\circ}$

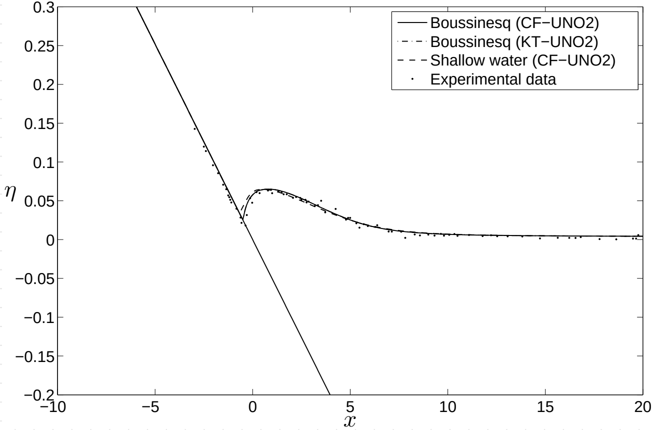


Figure: $t = 38$ s

Solitary wave runup: $A_s = 0.04$

Data from C. Synolakis (1987), $\beta = 2.88^\circ$

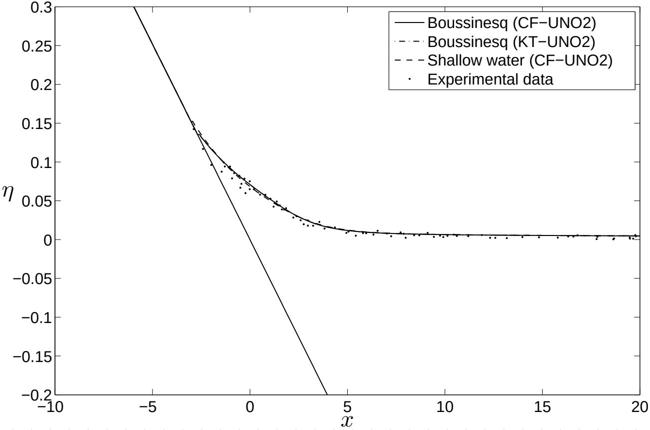


Figure: $t = 44$ s

Solitary wave runup: $A_s = 0.04$

Data from C. Synolakis (1987), $\beta = 2.88^\circ$

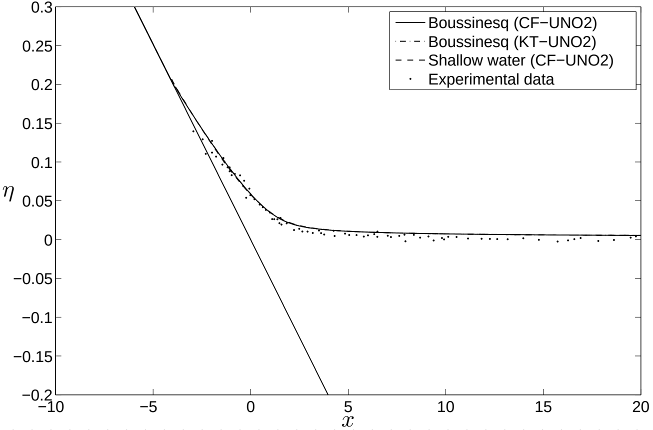


Figure: $t = 50$ s

Solitary wave runup: $A_s = 0.04$

Data from C. Synolakis (1987), $\beta = 2.88^\circ$

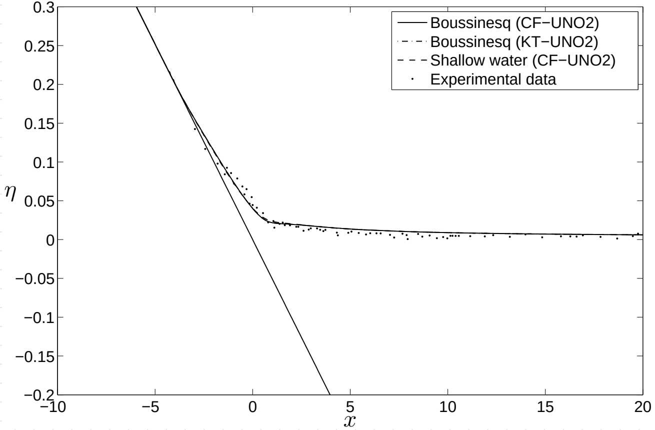


Figure: $t = 56$ s

Solitary wave runup: $A_s = 0.04$

Data from C. Synolakis (1987), $\beta = 2.88^\circ$

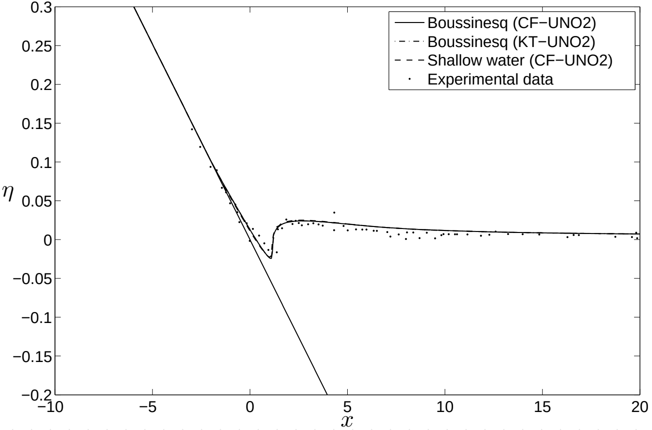


Figure: $t = 62$ s

Solitary wave runup: $A_s = 0.28$

Solitary wave runup: $A_s = 0.28$

Data from C. Synolakis (1987), $\beta = 2.88^\circ$

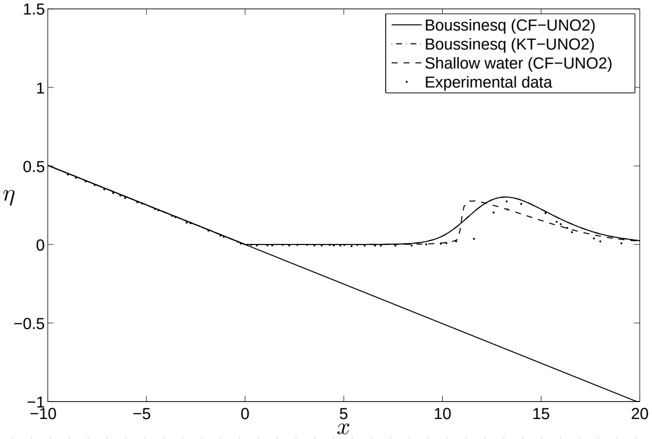


Figure: $t = 10$ s

Solitary wave runup: $A_s = 0.28$

Data from C. Synolakis (1987), $\beta = 2.88^\circ$

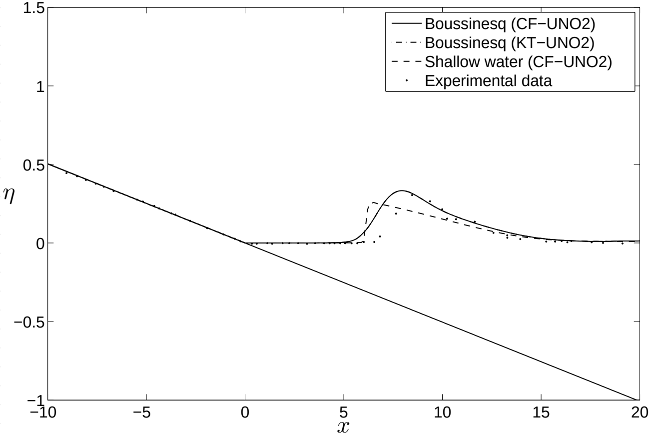


Figure: $t = 15$ s

Solitary wave runup: $A_s = 0.28$

Data from C. Synolakis (1987), $\beta = 2.88^\circ$

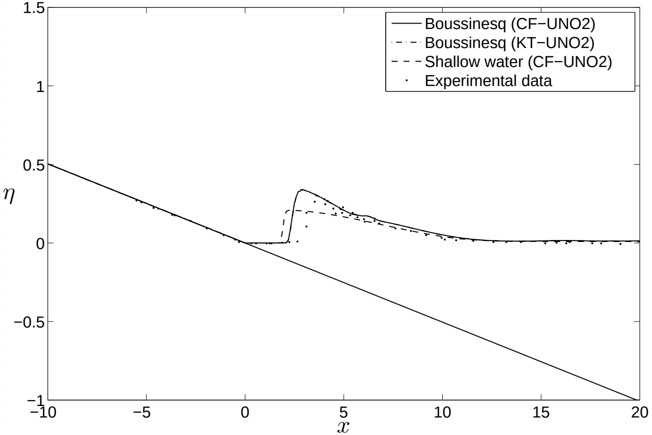


Figure: $t = 20$ s

Solitary wave runup: $A_s = 0.28$

Data from C. Synolakis (1987), $\beta = 2.88^\circ$

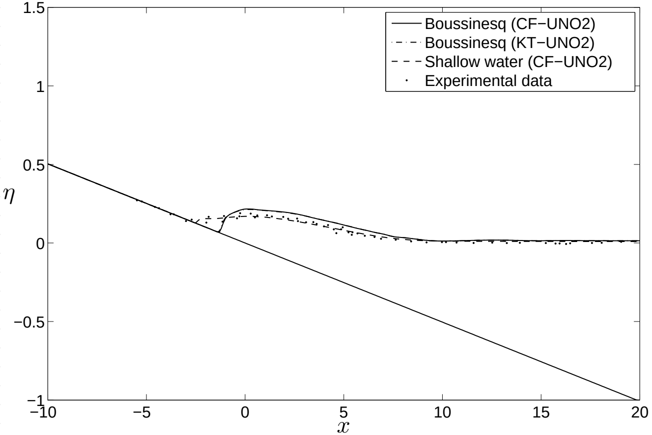


Figure: $t = 25$ s

Solitary wave runup: $A_s = 0.28$

Data from C. Synolakis (1987), $\beta = 2.88^\circ$

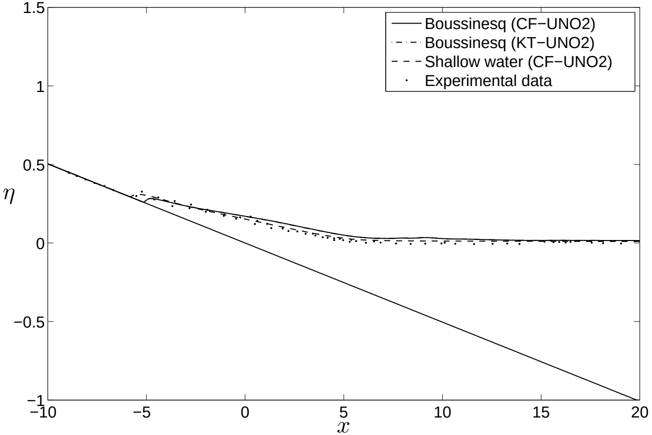


Figure: $t = 30$ s

Solitary wave runup: $A_s = 0.28$

Data from C. Synolakis (1987), $\beta = 2.88^\circ$

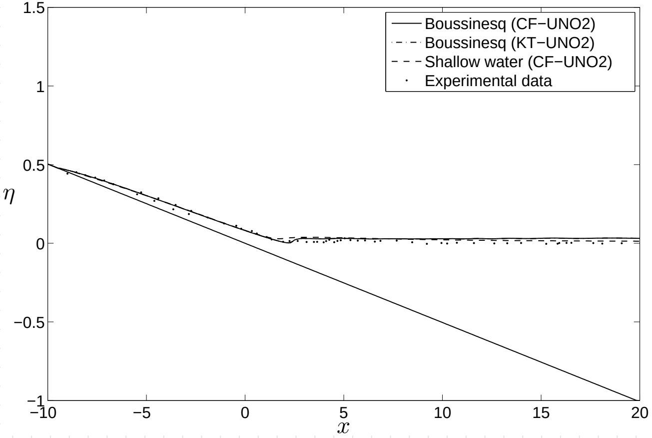


Figure: $t = 45$ s

Solitary wave runup: $A_s = 0.28$

Data from C. Synolakis (1987), $\beta = 2.88^\circ$

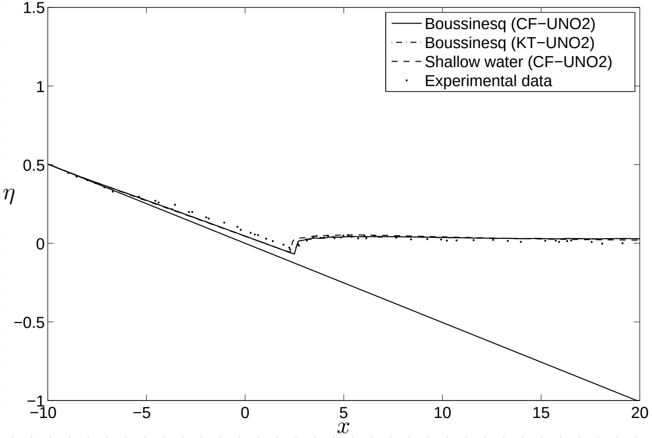


Figure: $t = 55$ s

Solitary wave runup: $A_s = 0.28$

Data from C. Synolakis (1987), $\beta = 2.88^\circ$

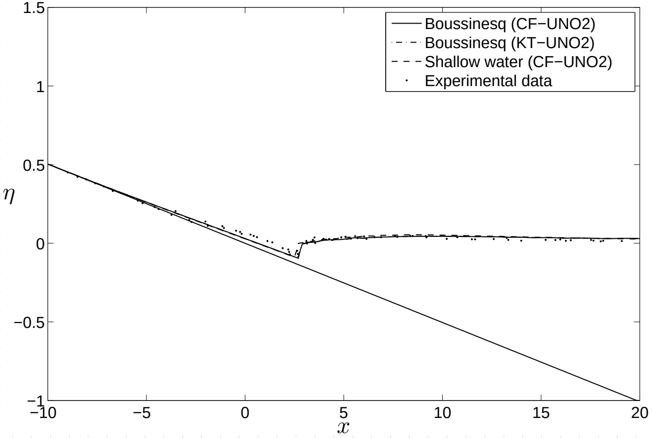


Figure: $t = 60$ s

Solitary wave runup: $A_s = 0.28$

Data from C. Synolakis (1987), $\beta = 2.88^\circ$

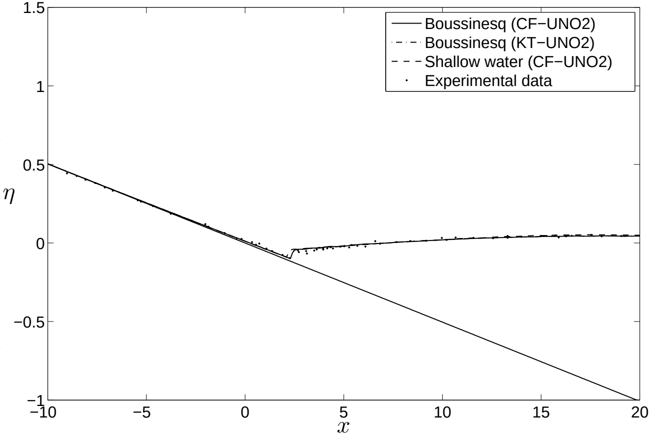


Figure: $t = 70$ s

Solitary wave runup: $A_s = 0.28$

Data from C. Synolakis (1987), $\beta = 2.88^\circ$

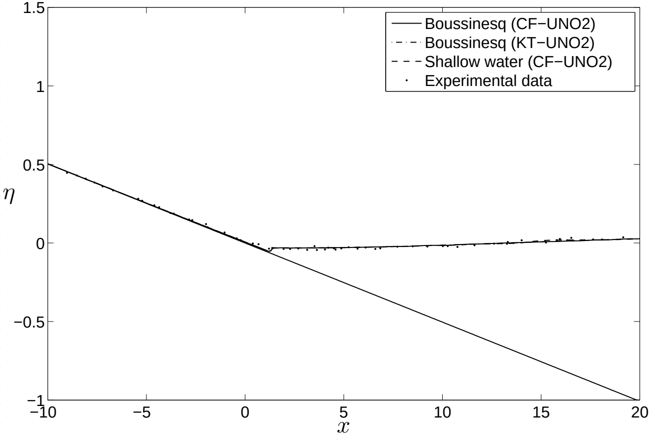
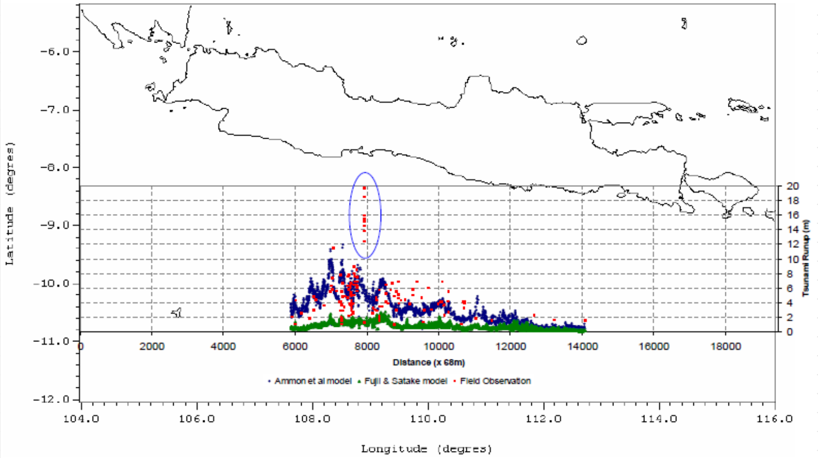


Figure: $t = 80$ s

July 17, 2006 Java Tsunami

By courtesy of Widjo Kongko (FI-LUH, Hannover)

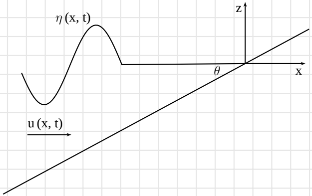


How to explain extreme runup values?

Simple academic test-case

Monochromatic wave runup

- Left boundary condition: $H_0(t) = d_0 + a_0 \sin(\omega t)$
- Incoming periodic monochromatic wave



Reference:

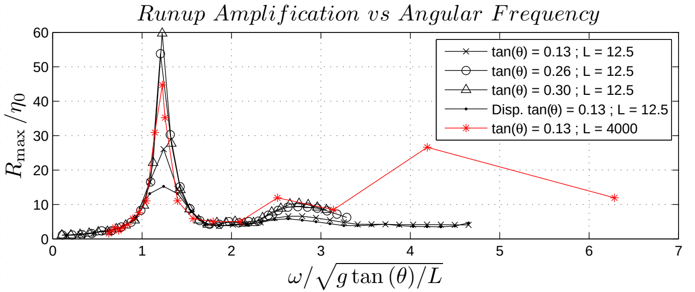
I. Didenkulova, E. Pelinovsky. *Run-up of long waves on a beach: the influence of the incident wave form*. Oceanology, **48**, 2008

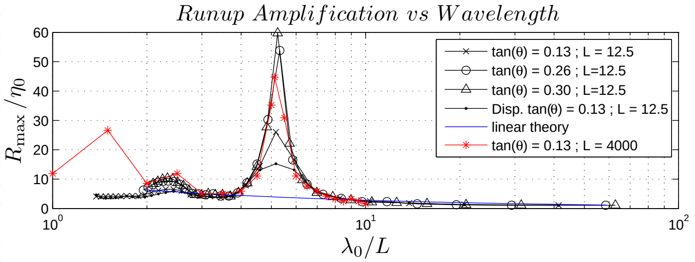
- Linear theory was shown to predict correctly at least the maximal runup
- $R_{\max} \sim \sqrt{\omega}$

- We compute numerically the $R_{\max}$ for various values of ω

Runup amplification

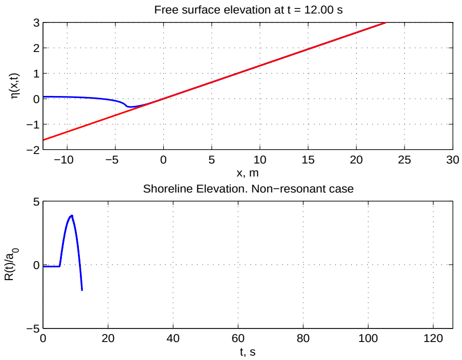
Constant sloping beach: some analytical considerations by K. Kajiura (1976)





Runup amplification: non-resonant interaction

Numerical illustration of a non-resonant case

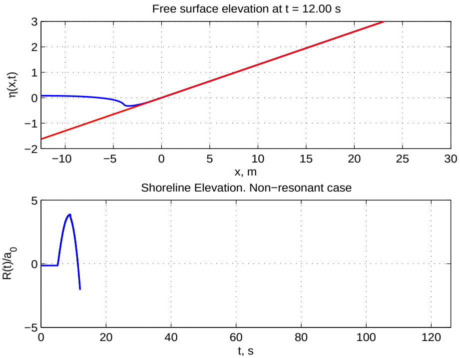


Reference [SDD11]:

T. Stefanakis, F. Dias, D. Dutykh. *Local Runup Amplification by Resonant Wave Interactions*. Physical Review Letters, 2011

Runup amplification: resonant interaction

Numerical illustration of a resonant case

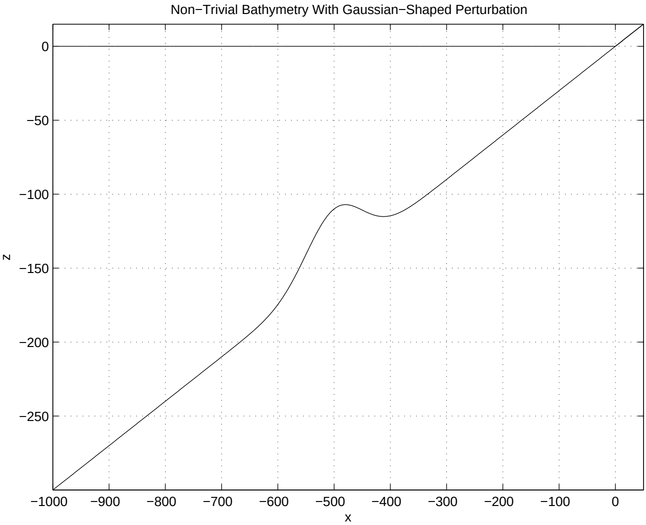


Reference [SDD11]:

T. Stefanakis, F. Dias, D. Dutykh. *Local Runup Amplification by Resonant Wave Interactions*. Physical Review Letters, 2011

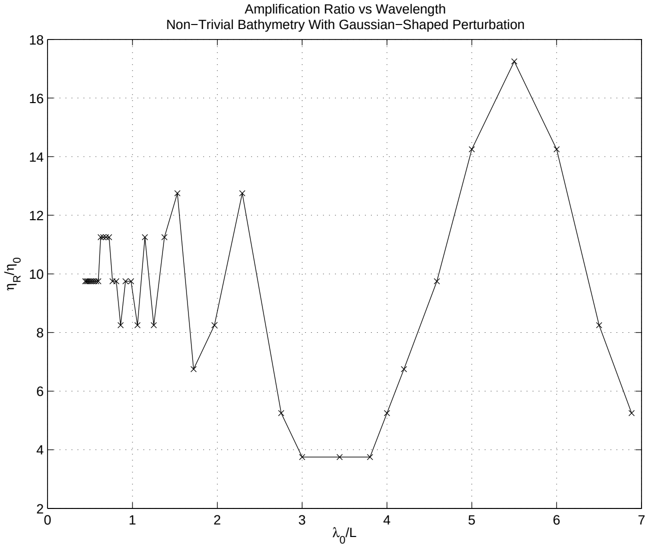
Non-trivial bathymetry - I

Gaussian bump on a sloping beach



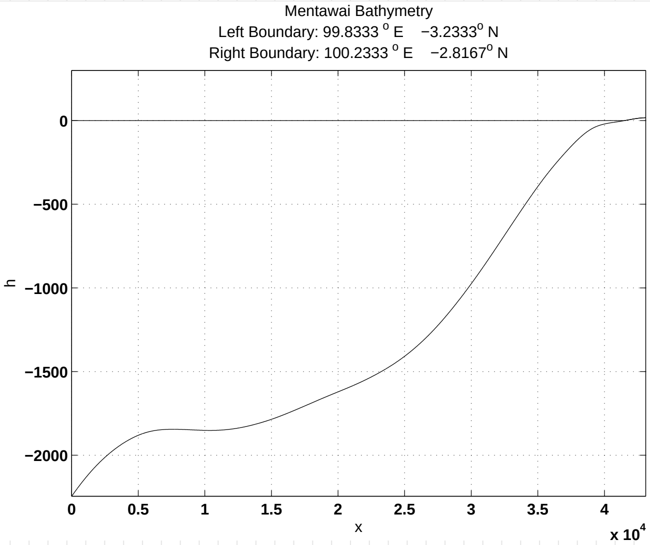
Non-trivial bathymetry - I

Gaussian bump on a sloping beach



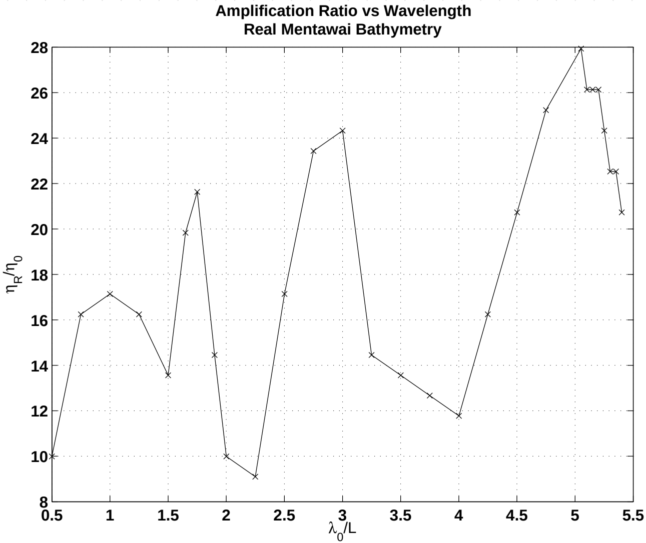
Non-trivial bathymetry - II

Mentawai Island bathymetry slice



Non-trivial bathymetry - II

Mentawai Island bathymetry slice



Conclusions

Principal points of this talk:

Numerics:

- A numerical method to discretize dispersive wave equations was proposed
- Dispersive effects are beneficial for the description of breaking waves

Physics:

- Wave resonance mechanism was illustrated
- Amplification of the maximum runup value on general bottoms was studied

Thank you for your attention!

http://www.lama.univ-savoie.fr/~dutykh/

References I

- D. Dutykh, Th. Katsaounis, and D. Mitsotakis, *Finite volume schemes for dispersive wave propagation and runup*, Journal of Computational Physics **230** (2011), 3035–3061.
- D. H. Peregrine, *Long waves on a beach*, J. Fluid Mech. **27** (1967), 815–827.
- T. Stefanakis, F. Dias, and D. Dutykh, *Local runup amplification by reso-
nant wave interactions*, Accepted to Physical Review Letters `http://hal.archives-ouvertes.fr/hal-00605461/` (2011).
- R.M. Sorensen, *Basic coastal engineering*, Springer, 1997.
- C. Synolakis, *The runup of solitary waves*, J. Fluid Mech. **185** (1987), 523–545.